IMAGE SYNTAX, PROOFS AND DIDACTIC APPLICATIONS

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The capacity for abstraction and concentration has changed. Methods of teaching Probability and Statistics must also evolve. Following is a presentation of an organized set of images, analysed using Graphs Theory, specifically adapted by syntax to each type of information. We use these, not as an illustration but as an exhaustive proof. We make this claim after having established the isomorphism between possibilities and elementary surfaces. Probability and statistics are thus unified and simplified. Combinatory Algebra and Mathematical Analysis remain tools. No more theorem will need to be learnt by heart. Fear and mathematical inferiority will be changed into an intellectual comfort. We have been using this teaching method for more than 25 years, in a basic course for non-specialized students, having written the corresponding books. Clinical and statistical tests already show its efficacy.

INTRODUCTION

Statistics and Probability Calculus had very different history until the second half of the 19th century, when Adolphe Quetelet attempted to mix these 2 disciplines, creating Mathematical Statistics. Anybody who seeks to understand this field must contend with the inherent difficulties of Statistics and Probability as well as the relationship between them.

During the second half of the 20th century, computers were developed, that simplified calculations, enabled us to perform more complex operations and even changed our psychology. Today everyone is accustomed to dealing with an infinite number of constantly changing images and most of us have no desire to spend long periods of time on abstract thinking and calculations. We are all in a hurry. A new, simpler method of teaching Statistics and Probability would certainly have much to offer to our contemporary world (Agrell & Berrondo-Agrell, 1992).

For this purpose, we present here, after *a short Graph Theory vulgarisation, an image set, with its syntax, its validity proof and didactic application.* it should be noted that our syntax consists of the method to transform a statistic or probabilistic data into an appropriate graph (Generalized Diagram), work on it in the simplest way, and come back to the initial data for giving the right solution.

BASIC GRAPH THEORY VULGARISATION

The eyes of a mathematician can look at a drawing from the Euclidian perspective (classical geometry), or from the point of view of Euler (graphs theory). Let us present elements of Graphs theory in a way that non-mathematicians, and even children, can understand. Let us consider any kind of drawing, such as that presented in Figure 1, for example.

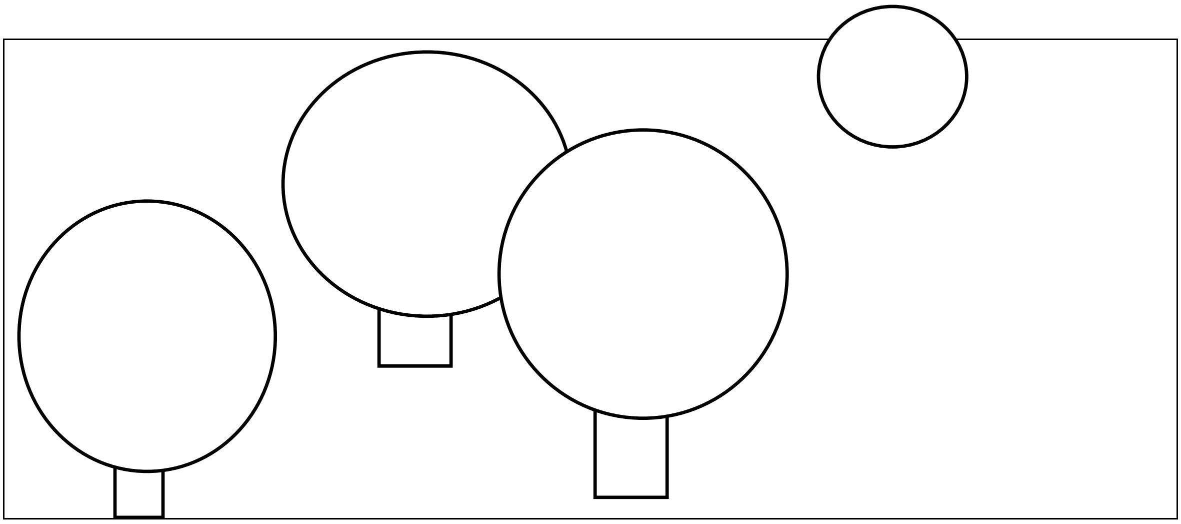


Figure 1. Any Kind of Drawing.

A drawing is a set of lines attached to big points in each of its 2 extremities.

If we were using scissors instead of a pencil, we should obtain several pieces of paper, elementary surfaces: 7 + 1 = 8 (without forgetting the main piece of paper itself). Number of elementary surfaces: s=8. The number of big points is: b = 15.

The number of (straight or curved) lines between points is: l = 19.

The number of connected sets is the minimal number of piercing the paper we have to do to cut with scissors along all the lines is: c = 3 (one for the isolated tree, one for the couples of trees, one for the sun).

Euler's formula says: s + b = c + l + 1. Let us verify it here: 8 + 15 = 3 + 19 + 1

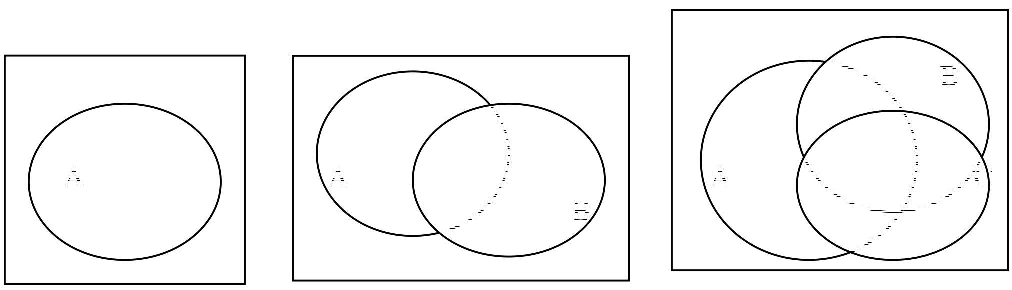
Thanks to this vulgarisation, the *links between drawings and mathematics* become obvious, and the *area concept* can be used in the following paragraphs.

AN IMAGE SET

Probability and Statistics are based on reality, which means real data systems: logical one, quantitative one, qualitative one or plural. We shall present here corresponding images for each of these data system, where possibilities correspond to elementary surfaces, by isomorphism , with U operation.

1-LOGICAL DATA SYSTEM "direct diagrams"

When the data is an ordinary set of subsets (logical information), we'll use classical Venn diagrams (Grunbaum, 1984; Venn, 1881) until level 3: (refer to Figure 2).



One logical data	Two logical data	Three logical data
2 elementary surfaces	2^2 = 4 elem. surfaces	8 elementary surfaces

Figure 2. Classical Venn Diagrams.

For more than 3 subsets, we'll use the Anthony Edwards (1989) infinite representation method that we can see in Figure 3 from level 4 to level 6.

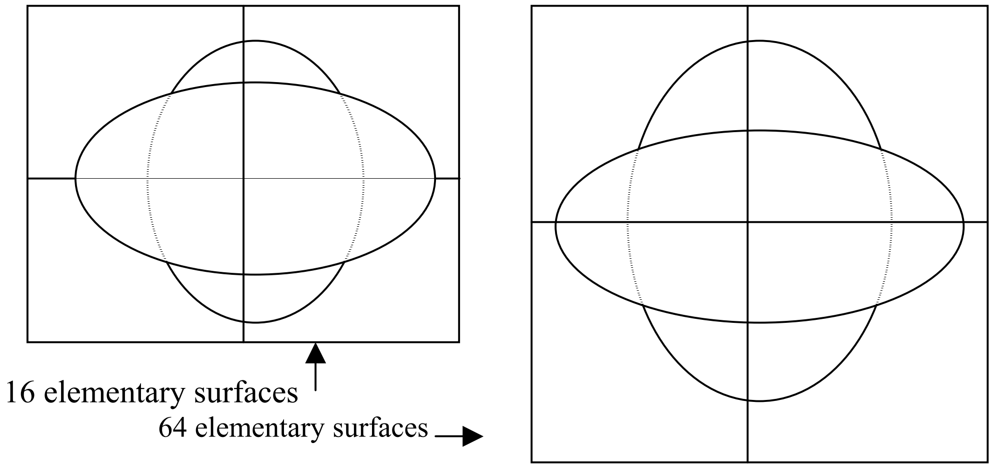
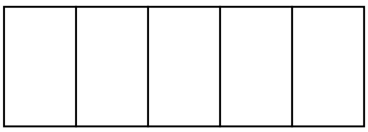


Figure 3. Anthony Edwards (1989) Infinite Representation Method.

Computers can continue these more and more complex teeth wheels, cutting our basis rectangle into 2 elementary surfaces, whatever n is.

2 – QUALITATIVE, QUANTITATIVE OR PLURAL DATA SYSTEMS

When the data is a partition (qualitative, discrete quantitative, or intervals quantitative information), we'll use *a simple rectangle divided into columns* or *an ordinary matrix* if this data is also plural (as shown in Figure 4). We call these **"**Partitive Diagrams**"**. With the previous "Direct Diagrams", they will create the set of "Generalized Diagrams" or G.D.



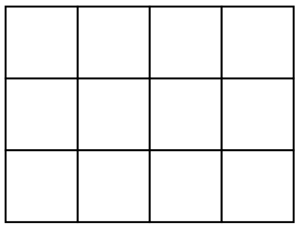


Figure 4. Partitive Diagrams.

3- ADDING A MEASURE WITH POSSIBLE SUPPLEMENTARY VARIABLES

We can add a positive real number on each elementary surface of a diagram. We call these diagrams: "Completed Generalized Diagrams" or "completed G.D.". We can also add a supplementary variable X. We call these diagrams: "Overcompleted G.D.". (There can even be several supplementary variables). Examples are given in Figure 5.

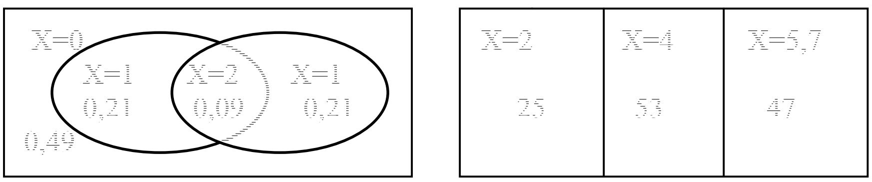


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Figure 5. Examples of Generalized Diagrams.

We'll use all these diagrams for a simpler pedagogy. The corresponding method has to be mathematically correct. Let us study it now

PROOFS

Let us consider a statistic, probabilistic or set theory context. Regarding the type of data, we chose a specific diagram, following the described syntax: each possibility is represented by a

specific elementary surface. Any proof concerning basic set theory, probability basic formulas or the understanding of links between statistics and probability, can be made in the easiest way by graphic method, thanks to our Generalized Diagrams. Indeed, *no possibility is forgotten* (exhaustive proof), and measure *theory axioms are automatically applied* by analogy with areas, that all human beings feels natural. Inside the rectangle (see Figure 5), everybody observe:
Non white area = SW-NE Area + SE-NW area – grid area

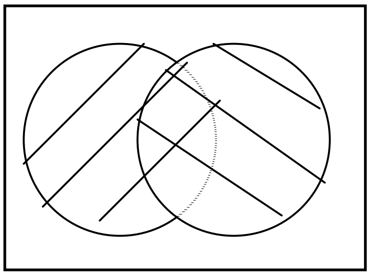

Figure 5. Non White Area = SW-NE Area + SE-NW Area – Grid Area.

Our representation syntax is based on a bijective application from the set of subsets of possibilities into the set of subsets of elementary surfaces, isomorphic with U, and respecting complementation (all this is easy to prove mathematically). *Respecting the natural area statement on the diagram, is respecting the basic measure axiom among the represented subsets. In particular, respecting the natural area statement on the diagram, is respecting the basic probabilistic axiom among the represented events.* Concerning *statistics*, there is *no more ambiguity between frequencies and statistics variables* (frequent source of mistakes for beginners), *links with probabilities get simple*, which is a *great help for test theory*.

DIDACTIC APPLICATION AND CONCLUSION

We have now established that apparent statements on the diagram are real statements among the original subsets. To benefit from the corresponding simple pedagogy (Fourastié & Berrondo-Agrell, 1992), we will not bother the students with the details of the mathematical proofs. The remaining difficulty is to find the right diagram (that respect the syntax).

We can compare this easy pedagogy to the traditional method of teaching addition and multiplication to young children. They use a simple way of getting the right solution, while remaining blissfully ignorant of the arithmetical sophisticated reasons why it works! As for the concrete application of the method in the teaching environment, we have used it for over 25 years to non-mathematicians (including children). We have obtained a significant number of clinical and statistical observations, showing how much easier it is to understand Boolean Algebra, Logics, Probability Calculus and Statistic using our approach.

This approach can even make the science of Probability-Statistics fun (Eurêka, 2000) rather than an abstruse, complex discipline reserved for an elite. As Emile Borel wrote (1926): “The calculation of probabilities is one of the most attractive and least difficult disciplines in the field of mathematics”.

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