MISCONCEPTIONS IN PROBABILITY

Li Jun
East China Normal University, China
Lionel Pereira-Mendoza
National Institute of Education, Singapore

The study investigated the probabilistic misconceptions of Chinese students, and whether selected misconceptions could be overcome through a focused teaching intervention. A questionnaire was given to a 567 Chinese students from grades 6, 8 and 12 and two streams (advanced and ordinary). In addition 64 of the students were interviewed. Fourteen groups of misconceptions were identified. The SOLO taxonomy was used in this study to describe students' hierarchical understanding levels on the concept of probability. It was found that, generally there was no improvement in developmental level from grades 6 and 8, the two grades without any formal probability training. Grade 12 students have a better understanding than the younger students. The results of the activity-based short-term teaching programme with grade 8 students show that even a short intervention can help students overcome some of their misconceptions.

INTRODUCTION

There is an extensive literature on misconceptions in probability. This literature identifies many misconceptions such as the equiprobability bias (Green, 1982; Fischbein, Nello & Marino, 1991; Lecoutre, 1992; Williams & Amir, 1995; Fischbein & Schnarch, 1997; and Cañizares & Batanero, 1998), the outcome approach (Konold, 1989, 1991; Williams & Amir, 1995) and representativeness (Tversky & Kahneman, 1974; Shaughnessy, 1977; Green, 1982; Konold, 1989; Fischbein, & Schnarch, 1997). These are only three examples of misconceptions, but illustrate the extensive nature of the literature.

While the literature provides ample evidence of the existence of misconceptions, the research has generally be undertaken in Western countries, where probability is an integral part of the school curriculum. This study extends this knowledge to a different cultural environment, namely China. In addition to the culture, in China probability is not introduced in schools, except in selected areas such as Shanghai, and even there it is limited to a very brief unit of approximately 8 hours in grade 12. The approach taken in this unit is theoretical, so students have no exposure to an experimentally-based approach to teaching probability. Consequently, this study extends the currently knowledge to both a different culture as well as a group of students who either have non-experience (grade 6 and 8 students) or have very limited experience (grade 12 students) of learning probability in school.

SAMPLE AND METHODOLOGY

The sample for the main study consisted of 567 Chinese students in grades 6, 8 and 12 (from both ordinary and advanced schools). They were given a questionnaire (9 versions involving parallel selections from 83 items) and in addition 64 students were interviewed and the interviews audiotaped. The items covered four categories: Category I - Identification of impossible, possible and certain events; Category II - Interpretation of chance values; Category III - Chance comparison in one-stage experiments; and Category IV - Chance comparison in two-stage experiments. The questionnaires were designed to identify misconceptions as well as to investigate the role that context and data played in eliciting misconceptions. The format for the 83 items was multiple-choice plus explanation. The response to an item was only coded as using a particular misconception when a clear answer and a consistent reason were given. The information obtained in interviews was used in analysing students' misconceptions but not used in counting the number of students having the misconception. Therefore, given the conservative nature of the coding the data presented here should be a lower bound for the actual number of students with a misconception.

The second part of the study involved a teaching experiment (six 40 minutes lessons twice a week outside regular classtime) with two grade 8 classes (25 and 26 students) to investigate whether focused instruction can help overcome misconceptions. The teaching was activity-based combined with whole class discussion. The activities were designed based on the information obtained from the main study. The intervention focused on the misconceptions related to identification of impossible, possible and certain events and the frequentist definition of probability. One class had access to a computer for simulations while the other did not. However, most of the conditions such as the activities, workbook, the problems for whole class discussion and the teacher were the same for the two classes. The main difference was that after students completed activities with dice, coins, and so on, in the computer lab class the data for a long series of experiments was simulated in front of the students, while in the other class the students were given the data for a long series of experiments and were told that they were computer generated. The instruments used in the pre- and post-test were parallel. Most of the items were selected directly from the 83 items developed for main study. All the students were interviewed before and after the two tests and all the interviews were audiotaped.

RESULTS - MAIN STUDY

Many misconceptions were observed in this study, and they were combined into fourteen groups: (1) subjective judgements; (2) example-based interpretations for possible and impossible; (3) possible means certain; (4) chance cannot be measured mathematically; (5) equiprobability; (6) outcome approach; (7) one trial is unrelated to other trials; (8) interpreting chance by data matching or word matching; (9) increasing repetitions is not better for predicting; (10) positive and negative recency; (11) used own methods in chance comparison; (12) taking different order as the same; (13) misuse or extend conclusions inappropriately; and (14) used own methods of chance calculation. The outcome approach, chance cannot be measured mathematically, compound approach and equiprobability were the main misconceptions for each grade and each stream of student. The results for these misconceptions is summarised in Table 1.

The meaning of the compound approach needs to be explained as it is used by the present authors and is not part of the accepted terminology in the literature. In solving multi-stage chance comparison problems many students tended to split a multi-stage experiment into several distinct one-stage experiments; then they compounded the results for each stage intuitively without any calculations. For example, consider the two-stage experiment of drawing one marble from each of the two bags, each of which contains some black marbles and white marbles. If in each bag there are more black marbles than white marbles students using this misconception believe that since drawing a black marble is more likely for each bag then drawing two black marbles is most likely for the two-stage experiment. Such a strategy used in chance comparison is referred to as the compound approach.

Table 1
Summary of Main Misconceptions for each Grade and Stream of Students

The four main misconceptions (Misconception used at least once)	Grades			Streams	
	Gr 6	Gr 8	Gr 12	Ord	Adv
Outcome approach	43%	39%	31%	38%	37%
Chance cannot be measured	46%	35%	19%	36%	29%
Equiprobability	20%	19%	18%	18%	20%
Compound approach	27%	17%	34%	24%	28%

It seems that except for the compound approach, the other three main misconceptions decrease with age. The fall is very obvious for chance cannot be measured mathematically, but the decline is marginal for equiprobability. This decline indicates that the misconception of chance cannot be measured mathematically can be overcome easily but the outcome approach and equiprobability are more stable. The compound approach occurred less often in grade 8 than in 6, but occurred much more often in grade 12 than in either grade 6 or 8. The increase in grade 12 might be because fewer grade 12 students thought chance cannot be measured mathematically and instead they thought that the compound approach worked in solving two-stage experiment items.

For the other groups of misconceptions, it was also found that subjective judgements and example-based interpretations for possible and impossible decreased with age. The two misconceptions belong to the lowest levels of understanding of probability. For most students, they can correct these misconceptions by themselves when they have more experience in dealing with uncertain situations. The misconceptions of interpreting chance by data matching or word matching and using own methods in one-stage chance comparison were used much less often by grade 12 students than grade 6 or 8 students. The misconception of increasing repetition is not better for predicting was used less often by grade 8 and 12 students than grade 6 students. In all, except for six groups where an analysis of the role of stream was not reasonable, most of the other eight groups of misconceptions were used less often or equally often by older students and advanced school students. Only the compound approach was used more often by grade 12 students and advanced students.

Context and data played a role in eliciting some misconceptions. For example, consider the results for the four main misconceptions. The use of chance cannot be measured mathematically does not seem to depend on data or context. The compound approach is influenced by data but not influenced by context. The approach was used more often when the compositions of bags/spinners were quite different from each other than they are equal or close. Actually, when the compositions were quite different from each other, the results from using the compound approach are in agreement with those that could be deduced from probability calculations. The outcome approach is influenced by context but may not be influenced by data (whether using a 50% chance elicits more outcome approach responses is still unclear). Both context and data affect the equiprobability bias. More equiprobable responses were observed in drawing names' items than drawing marbles or spinning arrowhead items, and also more equiprobable responses were observed when the compositions were close than when they were far apart. It may be that students' familiarity with drawing a name, where everyone has an equal chance (all names are equiprobable) leads to more equiprobability misconceptions. When the data are close some students use the specific version of equiprobability, namely that if the chances are close they are the same in practice, resulting in more equiprobability responses.

In addition to the identification of misconceptions, the level of sophistication of responses was analysed using the SOLO taxonomy. A cognitive framework indicating the developmental process of students' understanding of probability was generated (see Table 2). First, in general, there is no improvement in the developmental level between grades 6 and 8, the two grades without any formal probability training. Grade 12 students have a better understanding than the younger students. Investigating the data by streams it can be concluded that ordinary school grade 6 and 8 students had the poorest understanding of probability, advanced school grade 6 and 8 students had a slightly better understanding, but not as good as ordinary school grade 12 students. Advanced school grade 12 students had the best understanding of probability among all the students. Second, the variation between the students with and without formal training in probability appears to be less pronounced for category II items (interpretation of chance values) and more pronounced in category IV items (chance comparison in two-stage experiments). This was mainly because the training was focused on probability calculation and it enabled a large number of grade 12 students to answer category IV items at a higher level. However, all the students in this study had very limited experience with probabilistic experiments, so their response levels were generally lower when they were required to interpret chance in a frequentist approach. Third, even when students can give correct responses their reasons are often not very sophisticated. For grades 6, 8 and 12, the percentages of responses at the R or E level were 8%, 11% and 27%, respectively. One possible reason for this result is that many items can be answered without calculating probabilities and the calculation was not required in the items.

RESULTS - TEACHING INTERVENTION

Four of the major findings associated with the teaching are included here. First, an activity-based short-term teaching programme can help grade 8 students improve their performance in terms of giving correct answers. After the teaching, the percentage of correct answers increased from about 50% to about 80%.

Table 2
The Developmental Process of Students' Understanding of Probability

SOLO Level	Summary Description
P	Blank, fully irrelevant, illogical, egotistic answers or inability to become engaged in item answers.
U	Explains that probability just means may or may not happen and believes chance cannot be measured mathematically so chance comparison is impossible. Considers an incomplete set of outcomes in solving problem.
M	May consider all possible outcomes for a one-stage and sometimes for a two-stage experiment in qualifying uncertainty or estimating subjective chance value. For example, assigns an equal chance to each possible outcome for fairness. Interprets most likely to happen as meaning it should happen or interprets chance by frequency but without fully understanding the role of repetition. Uses rudimentary non-proportional reasoning in chance comparison.
R	Groups all possible outcomes in favour of a target event together and uses ratio as a measure of probability. Uses proportional reasoning in chance comparison. Knows that a larger number of repetitions is a more reliable predictor and expresses the idea of making a few repetitions automatically.
E	Assigns a calculated probability value in complicated situations, for example, involving two bags, two spinners and bases chance comparison on the values. Uses a generative strategy to construct sample space in a two- or three-stage experiment to work out probability. Suggests collecting data from a series of experiments and finding trends across sampling.

Second, there was a reduction in terms of the misconceptions used after the teaching intervention. All the fourteen groups of misconceptions except groups 13 (misused or extended conclusions inappropriately) reported in the main study were also observed in the teaching experiment. Equiprobability, chance cannot be measured mathematically, the outcome approach and own methods in chance comparison (the compound approach is one of the methods) were the most common misconceptions prior to the teaching intervention. Equiprobability, outcome approach, own methods in chance comparison and own methods in chance calculation were the most common misconceptions after the teaching intervention. Although, equiprobability, outcome approach and own methods of chance comparison were still the most common, they all occurred less often after the teaching intervention. The percentage of students who used them decreased from 61% to 43%, from 53% to 37% and from 35% to 31%, respectively. The misconceptions of chance cannot be measured mathematically, subjective judgements and example-based interpretations for possible and impossible were easily overcome. After the teaching intervention, the percentage of students who used them decreased from 59% to 12%, from 14% to 2% and from 31% to 4%, respectively. Third, there was an increase in terms of the developmental level of thinking as determined by the SOLO analysis of responses.

Finally, it appears that studying frequentist probability does not necessarily contribute to students' knowledge of classical probability. In the teaching intervention, the frequentist definition of probability was emphasised but the classical definition was not given. After the teaching it was found that some students could use proportional reasoning in chance comparison but actually they did not know the real chance values.

CONCLUSION

The following conclusions can be drawn (keeping in mind that they are based on the sample of about 600 students selected for this study): (1) Outcome approach, chance cannot be measured mathematically, compound approach and equiprobability are four main misconceptions of probability in Chinese students independent of school streams or background in probability; (2) Students' understanding of probability does not improve naturally with age - teaching plays an important role; (3) A short teaching intervention can help ordinary grade 8 students overcome some misconceptions and improve their understanding of frequentist definition of probability; (4) Introducing probability in the frequentist approach or in the classical approach cannot replace each other; each has its role in helping students' understand probability; and (5) The specific context and data involved in an item plays a role in eliciting some misconceptions. Given the limited length of this report detailed results can be found in Li (2000)

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