MATHEMATICS EDUCATION AND STATISTICS EDUCATION: MEETING POINTS AND PERSPECTIVES

Tânia Maria Mendonça Campos, Cileda de Queiroz e Silva Coutinho and Saddo Ag Almouloud
PUC-SP, Brazil
tania@pucsp.br

This article will discuss the research currently underway at PUC-São Paulo that represents links between Mathematics Education and Statistics Education. We shall discuss how these two areas of knowledge are entwined, verifying the advantages and consequences of these inter-relationships. The National Curriculum Parameters, which specify the Mathematics curriculum for the Brazilian school system, require one block of study involving the discussion of the themes Statistics, Probability, and Discrete Mathematics. These themes are considered throughout the entire 11-year period of basic education (students begin school at age 7) and also in the University. Research related to this problem is beginning to emerge, focusing not only on pre-service or in-service education of teachers, but also related to the learning processes of students, or to research interventions involving both teachers and students.

OPENING DISCUSSION
The study of statistical and probabilistic concepts starts, in the Brazilian school program, in the children's first years of study, accordingly with the National Curriculum Parameters for Elementary School launched on 1997. We shall begin this text with a brief explanation on the serial organization of the Brazilian school system.

Table 1: Organization of Basic School in Brazil

Level	Grade	Age (aprox.)
Elementary School I (EF-I: generalist professor)	1st	7
	2nd	8
	3rd	9
	4th	10
Elementary School II (EF-II: specialist professor)	5th	11
	6th	12
	7th	13
	8th	14
High School (EM: specialist professor)	1st	15
	2nd	16
	3rd	17

According to orientations in that document, PCN-EF, themes related to Statistics, Probability and counting problems should be worked out during the entire school period, in a progressive way. They are always mentioned in Mathematics classes and, occasionally, in transversal, interdisciplinary themes. Teachers responsible for such classes are not statisticians, but generalist teachers, eventually from Math graduation courses. But what can we say about that teacher's skill to work with the first contacts with Stochastics? Here we have a first interface between Statistical Education and Mathematic Education. The problems in teaching and learning stochastic concepts for Basic School (Elementary + High School) are worked, in Brazil, under the same precepts and theoretical basis than mathematical concepts, considering aspects of the teachers' formation as well as the student apprenticeship.

TEACHING AND LEARNING OF STATISTICS AND PROBABILITY
The National Curricular Directives (DCN) for Math courses (Licentiate and Bachelor), regulatory documents for university teaching in Brazil, do not specify any approach of Probability and Statistics, just quoting a phrase related to the Bachelor degree, after discriminating specific

contents, and referring to all Bachelor courses, whether in Pure or Applied Mathematics: "The common part must still include the study of Probability and Statistics." No mention, whatsoever, is made to the content discrimination for the licentiate courses, except an orientation such as "the contents of other areas related to Mathematics, which are sources for problems and application fields of such theories," should also be worked out. We believe the statistical and probabilistical thinking is included in such item, but nothing is explained in that document.

Research related to this problem start to emerge, searching to work the several problems resulting from the mentioned contradiction: on one side, PCN-EF indicating that contents on statistics and probability should be taught and, on the other, DCN not including specifically these themes in the teachers initial education, causing many institutions to reduce the time schedule dedicated to those contents or even to eliminate them from the curriculum.

Teachers working nowadays have searched in continuous education courses the resources to work with such themes, because they do not feel prepared or assured to include them in their teaching practice. School books, in spite of presenting chapters focused on the study of statistical figures or the first notions of probability do not perform it in a regular way, in most cases. Many times, we find such chapters in only one series, or then at the 5th series of Elementary II and later at the 8th series. With that, in-service training teachers do not have a supporting bibliography to prepare their classes and, even if they had taken Statistics classes in their academic education, they do not feel secure to work those contents with their students. Our experience with projects for in-service training teachers of the São Paulo state network allow us to witness such requests to include Statistics in the regular education courses. The following testimony, made at a virtual discussion forum created to support a course on Topics of Discrete Mathematics, in the Professional Master Degree Program of Post-Graduate Studies (Master Programs and PHD program) in Mathematics Education, at the PUC-SP, is quite illustrative about that:

> *Student X: I think we could start with a curriculum that gives more attention to the discipline, to information analysis and to the appropriation of information technologies as tools for teaching statistics. I remember that in University we used to be very sticky to producing tables and figures but we did not spent too much time on the analysis of the results, which is very important. As for the use of a computer, we did not even consider that hypothesis. As a teacher, I definitively felt the need of a continuous education.*

In our practice we have been able to observe that many teachers share this same type of anguish, this same concern for the work with the themes mentioned above. Licentiate courses on Mathematics include, in their majority, Statistical contents, though through an approach privileging the study of mathematic tools. Such approach does not allow the construction of a statistical reasoning, or that the students build a critical analysis from the calculation of summary statistics, involving variability around the average or around the median. In the PCN-EF we can find an affirmative: nobody teaches what he or she does not know. Well, if the teachers have not been prepared to perform such reading of the data, how can they teach that to their students? Some of the researches performed in the Master and PHD Degree Program of Post-Graduate Studies in Mathematics Education at the PUC-SP are already trying to discuss the subject, worrying about the aspects of teaching statistical contents in the Basic School and also in the Licenciate courses on Mathematics and Pedagogy.

RESEARCH PERFORMED AT THE PUC-SP

Since their first papers were concluded, researchers of the Master and PHD Degree Program of Post-Graduate Studies in Mathematics Education at the PUC-SP have been searching for their themes not only among mathematical contents, as properly said, but also among themes connected to the development of stochastic thinking and reasoning. Such works are aimed to the Basic School, as well as to the University, being that for teachers' education (in-service or pre-service teachers) or also for professionals that use statistics as a working tool. We now present concisely some dissertations and thesis developed within our program that are somehow related to the proposed themes, aiming at gather them according to common fields or focus (teacher or student).

Researchers Focused on Basic School Students

COUTINHO, C. Q. S. (1994). *Introduction to the concept of probability by a frequency point of view: an Epistemological and Didactical Study.* Master's Degree Dissertation. The aim of this research was to study spontaneous or pre-constructed conceptions from the students about chance and probability, analyzing the experimental sequences when introducing these concepts, starting from the observation of the stabilization of an event's relative frequency, after a great number of repetitions of the random experience. The final objective of the choice for a frequentist approach is undoubtedly to extend the probability notion not just to an "equally probable cases" situation, according to Laplace enunciate in his second principle, in "Philosophical Essay of Probability," but also to model complex situations such as questions of reliability and diffusion in oil research or stochastic control. As for a didactic objective, it is to deeply connect the teaching to the learning conditions in which the student is inserted. The results reached by the author worked out as an incentive for the introduction to the probability concept under the frequentist approach, but also considering the classic or Laplacian approach. The results also indicate the advantages in anticipating that introduction to the initial series of EF-II, and not just to High School.

SILVA, I. A. (2002). *Probabilities: the Laplacian vision and the frequentist vision in concept introduction.* Master's degree dissertation. From its origin, the concept of probabilities grew in multiple perspectives: concretely, the probability of an event or phenomenon has been conceived from a classic or laplacian view-point (based on the "Law of Laplace"), with an approach based on the relative frequency of the event in study (based on the "Law of the Great Numbers" of Jacques Bernoulli) and with a subjective approach. This master's degree dissertation had as objective the study and the application of a didactical sequence in which the concepts or notions that lead to the definition of probabilities could be approached starting from activities or situation-problem; the frequentist and the classic approach of probability could be integrated in the teaching process in order to have a deeper and more significant learning. The results of this research testify the possibility to build the notion of an event's relative frequency – and, therefore, the frequentist definition of probabilities – without necessarily conducting experiments in class. The students have achieved, according to the author, a study on the probabilistic theory with a broader and more significant approach of probability concepts: the laplacian and frequentist approaches seem to have been incorporated by the students.

CAETANO, S. S. D (2004). *Introducing statistics in the initial series of Elementary School, starting from manipulative material: a teaching intervention.* Master's degree dissertation. LIMA, R. C. R. (2005). *Introducing the concept of Arithmetic Mean in the 4th grade of Elementary School by using the computational environment.* Master's degree dissertation. The aim of these two dissertations was to investigate two groups of 4th grade children's skills in reading and interpreting graphs, as well as their conceptions of the "mean," one with the assistance of computational environment by using the Tabletop software, and the other with manipulative material instead of computer environment. The analysis of the results indicated students' difficulties in reading and interpreting graphs in particular situations, such as graphs with non-unitary scales and/or with null frequencies. The reading and the interpretation of two-entry graphs was not associated with major difficulties. For the arithmetic mean, an increase of almost 50% in the students' performance was observed from comparisons of the pre and post-test results. On the basis of these results, it appears that the teaching intervention enabled the development of strategies to solve the presented situations, as well as favouring the development of important relations between the concepts investigated, which influenced the amplification of the students knowledge on "information treatment." As for the group working on computational environment, it is possible to conclude that the introduction to the arithmetic mean concept based on graphic representation has been favoured by the use of Tabletop software, since it allows the students to catch the proprieties and the relations involved in the conceptual field formed by graphs reading and interpretation, as well as arithmetic mean.

STELLA, C. A. (2003). *A study on the concept of "mean" with High School students.* Master's degree dissertation. The aim of this study is to identify interpretations of the concept of arithmetic mean held by High School students following the Brazilian curriculum. With this aim in mind, the author investigates the characteristics of the concept of mean that are emphasized in

the High School Mathematics Curriculum and in research studies related to an understanding of the teaching and learning processes associated with this concept. The results of this research indicate that students perform well when dealing with problems involving weighted means and with problems in which they construct distributions of data sets. They indicate also that most of the students interviewed see the mean as an algorithm and have difficulties in solving problems in which they have to calculate the mean on the basis of graphically presented data.

Researchers Focused on Basic School Teachers

SANTOS, S. S. (2003). *The education of a non-specialist teacher, concerning the elementary concepts in Information Treatment: a case study in computational environment.* Master's degree dissertation. The aim motivating this dissertation was to investigate the possibilities offered by the computational environment of the Tabletop software in the formation process for elementary knowledge of statistics in teachers of the Elementary School I. The results suggest that to collect data and to organize data in a table or graphs represent an important stage in the study of a database and that the computer, particularly Tabletop, can contribute in a significant way to the understanding of graphs and tables extracted during these data manipulation. However, at the same time, the software Tabletop can help in the learning process of statistics, an inadequate use of any particular resources can hinder the study of some aspects of data representation and the development of other concepts.

GONÇALVES, M. C. (2004). *Teachers' conceptions and the teaching of probability in Basic School.* Master's degree dissertation. The aim of this research was to identify the current Mathematics teachers' conceptions of probability and to verify if there is a relation between these conceptions and the ones that were used in the schools in the 1970s, 1980s and 1990s, by analyzing some school books and the answers obtained in a questionnaire. The analysis of the available information enabled the author to sustain the evidence that the teaching practice influences the conception changes, since teachers who had their basic education in the same period and teach at different levels have, themselves, different conceptions. It is also important to say that the four types of conceptions identified confirm the results presented in the bibliography review made by the author, particularly in a study of the probabilistic conceptions with teachers of EF-I in Spain.

Researchers Focused on University Students

FIGUEIREDO, A. C. (2000). *Conditional probability: an educational approach.* Master's degree dissertation. This work had for objective to study the introduction to the Conditional Probability concept, as well as to the concepts connected to the Law of Total Probability and the Bayes' Theorem studied at University. The author developed a teaching sequence considering the principles of the Didactical Engineering. Among other conclusions, the author highlighted that most students, when dealing with Conditional Probability, identified it as something different from the calculus of the Probability of Events Intersection. They also were capable to distinguish the calculation of P(A|B) from P(B|A), since they were presented through questions in natural language. However, when similar subjects were presented in symbolic language, many students experienced difficulties.

SOUZA, C. A. (2002). *Binomial distribution in university education.* Master's degree dissertation. Due to the important role of the binomial distribution in Statistics and to some known difficulties in the teaching-learning process of Probability, this work was developed in order to understand some aspects of the learning process of the binomial distribution and its relation to other probability distributions. The author organized and applied a didactical sequence to support the understanding of binomial distribution. The results showed that the students do not construct the meaning of the binomial distribution in order to use it when the problem to be solved is presented and need this distribution as a tool. The use the students make are more due to the didactical contract then to their knowledge about this distribution.

NOVAES, D. V. (2004). *The mobilization of statistical concepts: an exploratory study with students of a course on Tourism Technology.* Master's degree dissertation. The objective of this research was to analyze if the students of a Bachelor course in Tourism Technology can usefully utilize concepts and conceptions derived from their learning of Statistics to solve

practical problems in their work area, as well as to identify difficulties and types of mistakes made after that learning process. The research was conducted on six couples of students who had already taken Statistics classes. Since it was an exploratory study, all its results should be considered as a starting point to deeper studies on the concept construction by students using Statistics as a tool in several professional contexts.

MIGUEL, M. I. R (2005). *Teaching and learning the Poisson model: an experience in modeling*. PHD thesis. This thesis concentrates on the teaching and learning process of the Poisson Model, whose questioning refers to the use of the Mathematical Modeling, which steps and results will be considered for a didactic interaction, as well as in the interpretation of the acquisitions and mistakes made by the students. The author expected that pair-work, computer use and practical experiments, might favour the development of her project. A teaching sequence was elaborated, based on the Mathematical Modeling, and it was applied to a group of university students attending the second year of Electric Engineering and Computer Sciences. This research leads to the conclusion that not only the use of the Mathematic Modeling is favourable for that study, but it also made possible to analyse and to interpret all the modeling steps that were adapted and considered essential for the research purposes to be achieved.

CONCLUSION

We intend in this article to illustrate the interfaces between Mathematics Education and Statistical Education and the works made, so far, in the professional master degree program of post-graduate studies in Mathematics Education at the PUC-SP. Other works are actually under preparation, strengthening even more the relations between these two areas of knowledge, either by using a common theoretical pattern, or by studying the articulations between them that might lead at the development of a school curriculum. In the Basic School as well as in the College Level the study of statistical concepts or, even better, the concretisation of a statistical alphabetization depends on these interfaces, since mathematic tools are essential, but not sufficient, to build concepts in an operational and significant manner in this area.

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