# PROSPECTIVE TEACHERS’ AWARENESS OF YOUNG CHILDREN'S STOCHASTIC ACTIVITIES 

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The present research focuses on prospective teachers' planning, teaching and reflection on young children's (4 to 6 year-old) stochastic activities. The research also concentrates on the way in which prospective teachers view the activity in terms of the mathematical challenge it offers and the development of children's stochastic ideas (Potari and Jaworski, 2002). The methodology of this research is based on the qualitative approach. The researchers analysed twenty-three prospective teachers' lesson plans and actual teaching, interviewed them after their lessons and finally analysed their self-assessment reports. An initial analysis of the data shows that prospective teachers appreciated the importance of using tools in their classrooms for teachings stochastics. However, from our classroom observations we identified that the activity was often mathematically trivialized and the children's involvement was limited. The discussion and selfassessment that took place after the lessons indicated the different degree of prospective teachers' awareness of pedagogical and mathematical issues.

## THEORETICAL BACKGROUND

## Mathematics Teaching and Teacher Education

Mason (1998) states that awareness is a concept comprising both conscious and unconscious powers and sensitivities that enable people to act freshly and creatively in the moment. He considers that the key notions underlying real teaching are "the structure of attention and the nature of awareness." Schon (1987) names this knowledge as knowing in action. However, he highlights that this knowledge is not adequate on its own for teaching but teachers need to evolve through a progression from knowing - in- action to reflecting-on-action to reflecting-in-action. Reflecting- on- action occurs when the teacher starts looking critically at events after they have occurred while reflecting in action appears while teaching or planning for teaching. These stages involve a metacognitive awareness in which knowledge and action are linked. For analysing the complexity of teaching mathematics, Potari and Jaworski (2002) used the teaching triad (management of learning, mathematical challenge and sensitivity to students) as an analytical device and as a reflective agent for teaching development by teachers. Management of learning describes the teacher's role in the constitution of the classroom-learning environment by the teachers and students, for example planning of tasks and activity. Mathematical challenge describes the challenges offered to students to engender mathematical thinking and activity and sensitivity to students describes the teacher's knowledge of students and attention to their needs.

The mathematical activity is not only characterised by the content knowledge, but requires a broader view of knowledge like the mathematical know-how (Boaler, 2003). Noss and Hoyles (1996) and Hoyles (2002) claim that the mathematical activity is designed to foster mathematical meanings through construction, interaction and feedback, and also the students could scaffold their own thinking though communicating with the tools (emphasizing to the use of microworlds). However, these tools are often considered in early years more as a means to motivate children rather than to challenge them mathematically. One reason could be that the teachers have not constructed a flexible mathematical knowledge that allows them to plan activities that will encourage young children's mathematical activity. Moreover, they need to consider children's ideas and intuitions both in their planning and in their actual teaching. Shulman (1986) defines this knowledge as 'pedagogical content knowledge' and Ball and Bass (2000) believe that teaching mathematics entails work with microscopic elements of
mathematical knowledge in order to make sense of a child's apparent error or appreciate a child's insight.

## Learning and Teaching Probability in Early Age

The importance of doing mathematics from early age is related to the idea of constructing mathematics knowledge based on children's intuitions. Fischbein (1987) claims that intuitions develop as a kind of 'knowledge from experience,' which are used to take control over actions. He concludes that an intuition is the direct, cognitive prelude to action, mental or practical, which organises information in a behaviourally meaningful and intrinsically credible structure. Many researchers (e.g., Pitta-Pantazi and Tsamir, 2005) claim the importance of building mathematical concepts on intuitions. As experience plays the major role in early-childhood education, intuitions is the starting point for learning mathematics. Pre-primary teachers have the role to realise and build mathematical concepts on young children's intuitions.

Concerning stochastics, Paparistodemou and Philippou (2002) describe how young children start to make probabilistic decisions and think about chance and risk from an early age, depending on how they have embarked on probabilistic games. According to Borovcnik and Peard (1996), there is no doubt that the topic of probability is an important one in the mathematics curriculum even though the inclusion of probability is a relatively recent development. Nowadays, probability and statistics have an important role to play in our everyday life, especially at children, where most of their games have the idea of chance. Research on learning stochastics (Paparistodemou, 2004) has shown that young children use spatial representations for expressing stochastic ideas. Unlike Piaget's (1951, translated 1975) preoccupation with 'a priori' probability, Fischbein's perspective allows an exploration of intuitive foundations and precursors to probabilistic knowledge. Research (Paparistodemou, 2004; Paparistodemou and Noss, 2004; Pratt, 2000) also shows that the design of an activity at the age of 4-8 is very important for children to express probabilistic ideas. According to Borovenik and Peard (1996), there is no doubt that the topic of probability is an important one in the mathematics curriculum even though the inclusion of probability is a relatively recent development. A necessary area of investigation is the role of the teacher: if we consider the necessity of educating students who are used to think stochastically, it is needed to re-think the role of the teacher in the teaching/learning process. Konold (1991) provides some salient reflections on the role of instruction in a learning situation fraught with students' prior stochastic' misconceptions':
'... My assumption is that students have intuitions about probability, and that they can't check these in at the classroom door. The success of the teacher depends on how these notions are treated in relation to those the teacher would like the student to acquire... How students think about probability before and during instruction can facilitate communication between the student and the teacher.' (p.144)
Petocz and Reid (2002) indicate the importance for development of learning environments that can engage students' interest, broaden their understanding of statistics and enrich their own lives. They suggest that the development of learning environments must be 'total' and that the learning of stochastics should be less focused on the curriculum itself, and certainly less focused on the traditional concern of material to be 'covered' or 'examined.' Rather, the focus should move towards supporting students to develop 'holistically.'

The present research focuses on prospective teachers' planning, teaching and reflection on young children's (4 to 6 year-old) stochastic activities. It concentrates on the way in which prospective teachers realise the activity in terms of the mathematical challenge it offers and the development of children's stochastic ideas (Potari and Jaworski, 2002). Two research questions are addressed in this paper:

- What is the prospective teachers' attention when they plan, teach and reflect on their teaching?
- What is their mathematical awareness, which underlies their teaching decisions?


## METHODOLOGY

The methodology of the research follows a qualitative approach. The participants are a group of prospective ( 23 subjects) teachers. The prospective teachers were doing their teaching practice in pre-primary schools as a part of their university studies and the data comes from their lesson plans, observations of their teaching, interviews after their teaching and their written selfassessment reports. Each prospective teacher taught 8 lessons on different mathematical subjects to the same pre-primary classroom (age of children 4-5.5). The role of the researcher at the classroom observations was the one of the observer as a marginal participant (Robson, 1993) and the interviews were 'semi-structured' (Scott and Usher, 1999). Prospective teachers wrote their self-assessment reports after their teaching, where they expressed their thoughts concerning the positive and the negative aspects of their teaching. In the self-assessment reports teachers had the chance to reconstruct their lesson plan. The data were analysed by identifying segments related to the two research questions. In our attempt to respond to our first question, categories were formed to describe the teaching elements that the prospective teachers attended to. The second question was addressed through an analysis of the activities posed to the children and of the prospective teachers' reflections on these.

## FINDINGS

## Mathematical Challenge: Prospective Teachers’ Mathematical Awareness

The findings of the two research questions, what prospective teachers attend to and what is their level of awareness, will be discussed under the element of the teaching triad: mathematical challenge. In prospective teachers’ lesson plans we identify that teachers’ aims refer to mathematical challenge like the mathematical concepts of certain, impossible and probable events. From their activities we recognised that they did not seem to realise the concept of randomness and it appeared to be a difficulty to see from the task (the context) the mathematical point. For example, a prospective teacher, Evelyn (all names are pseudonyms), described one of her activities in her lesson plan as follows: 'I will tell the kids that the animals left there cells and ask them why this might have happened.' In her teaching she emphasised her activity on children to 'solve the mystery of the zoo' and children were using words like 'probably,' but although as she said the aim of the activity was for children to 'express verbally probability' there was an absence of randomness in this scenario. Other activities followed also with the absence of randomness, referring to what might have happened and what not.

In Amy's case we also have an example of activities with children to make a guess 'of what may be the drawing hidden behind a carbon by partially revealing it.' Her aim again was to express the word possible. However, students were not directed towards the mathematical content of probability, since again, the element of randomness was missing. Thus, although children might reason what the picture might be, they did not really approach the idea of probability. In the interview the researcher asked her to explain how she understood the probability ( R stands for the researcher):

R: What does the concept of probability mean to you?
A: The may be...that something is possible.
R : What does it mean when we say something is possible?
A: ...I know the word...it is at the edge of my tongue...but it doesn't come to me now...Well, not to know something...
R: ...Does possible mean guessing?
A: In this activity I did not want them to guess...but...I don't know....may be this particular activity did not refer to probability?...I do not know...What do you think?
We can argue here that the idea of probability was confused to 'what something may be' instead of 'what may happens.' Amy appeared not to realise that the probability is related to the likelihood of an event occurring and she related it with the likelihood of something being a certain object. In Amy's words we identified some 'apprehension' in her words. We could argue that the teacher based her definition to her own intuitions and she has not been able to identify the mathematical meaning of what a probability is.

In Macy's lesson plan we recognised an awareness of the concept of randomness. Macy was very specific in her two aims of the lesson, which were 'Children should be in a position to
select the appropriate sample spaces for certain, impossible and fair events' and 'Children should experience the concept of randomness.' In her first activity she was planning to show children 6 spinners, which had different sample spaces (for example, whole blue, whole red, half blue half red, $3 / 4$ blue and $1 / 4$ red etc.). Children would be asked question such as:
'Which spinner should the blue boy use in order to win? Why? If I spin this spinner will I get blue for sure? Why?'
The interesting point here is that she used the 'why' question and her activity was addressing the aims of the lesson. Macy seemed to be very aware of the epistemological characteristics of her activity in her lesson plan. Although in the planning of the activity she stated that children would experience the concept of randomness, when she was teaching she presented to the children the spinners and asked them what colour would get without spinning any of the spinners. In addition to this, her questions were often phrased to direct specific answers. For example, 'For blue to win shall we get the blue spinner?'. This had as a result children providing yes or no answers and not using the words certain, impossible, possible. Macy's case is interesting from the point of view that although we recognised from her lesson plan a mathematical challenge awareness, we can see from the her teaching that she was not aware of a deeper understanding of the probability concept and the presence of randomness in her activities. On this point, in her interview she stated:

M : This activity aimed at children seeing different sample spaces and making decisions in regard to which colour would win.
R: What did you want children to learn?
M: ...Different sample spaces...Well, the activity went well, so the level of difficulty was good and children did not develop any misconceptions.
We can recognise here that Macy used words like 'sample spaces,' 'making decisions' and reflects on her activity on mathematical issues. Besides that, we can say that there is a limitation on this mathematical challenge, as she did not recognise the importance of randomness on her probabilistic activities.

In their self-assessment reports some teachers seem to realise their mathematical limitations. Evelyn expressed her thought as follows:
'I feel that the lesson did not go so well for several reasons. Some of them are: the fact that I had to think about the activities by myself, the fact that I was not sure of the concept of probability myself, led to the lesson plan not being very good.
However, I feel that the lesson would have been better if the children were calm.'
Evelyn was aware that if she had a deeper understanding of the probability concept this would have helped her to improve her planning. Besides that, she also reflected on management learning to address what she would have changed on her lesson, after her teaching.

## DISCUSSION

One of the most important findings of this study is that the prospective teachers attend mainly to management and affective issues, and to a lesser extend to cognitive and mathematical aspects of the tasks. Quite often the mathematical challenge seems to be rather trivial both in the planning and in the classroom teaching. Although they state the mathematical aims of the lesson, in many cases these aims appeared to have certain limitations. For example, sometimes they were very general, emphasized procedures or disconnected from the designed tasks. One explanation for this appeared to be prospective teachers' lack of mathematical awareness. For example, in some of the stochastic lesson plans one of the aims was for children to use and understand the words "certain," "probable," "impossible." However, although both prospective teachers and children were using verbally these words in various scenarios, these scenarios lacked the idea of randomness. Furthermore, they often used rather closed questions and not investigative activities. Overall, prospective teachers had difficulty in identifying the epistemological characteristics of the tasks. We do not see a mathematical awareness of some important concepts like that of randomness. Even in the case of Macy case where we recognise an understanding of this concept this is rather unrelated to the other probabilistic concepts, so we could say that the awareness remains at the level of action (mathematical) and it does not indicate a greater degree of awareness, that of the awareness in discipline (Mason, 1998).

Summing up, this study shows the kind of experiences that guide prospective teachers' design, implementation and evaluation of teaching. It can be argued that prospective teachers build relations between theory and practice at a rather general pedagogical level. The transition and reflection to more specific mathematical and pedagogical issues appears to be a difficult endeavour. This calls upon special attention and reflection on behalf of mathematics teacher educators to tackle this problem.

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