LISTEN TO THE STUDENTS: UNDERSTANDING AND SUPPORTING STUDENTS' REASONING ABOUT VARIATION

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During a research project investigating understanding of variation students in a tertiary level introductory statistics course completed a questionnaire prior to, and at the end of, the course. This paper reports on interviews of selected students designed to determine whether more information could be gathered, and to identify those teaching and learning activities that assisted students to develop understanding. Prompting assisted students to develop better quality responses but cognitive conflict situations proved challenging. The diversity of activities identified by students as assisting development of understanding provides a challenge for educators in planning teaching sequences. Both educators and researchers need to listen to students to better understand the development of reasoning.

INTRODUCTION

Statistical reasoning, involving reasoning with statistical ideas and making sense of statistical information, depends heavily on the understanding of basic underlying concepts (Garfield, 2002) and one important underlying concept is variation (Wild and Pfannkuch, 1999). The recent research focus on reasoning about variation (Reading and Shaughnessy, 2004) has necessitated more qualitative research into levels of reasoning (e.g., Watson, Kelly, Callingham and Shaughnessy, 2003). This often involves coding of responses to open-ended questions, with related uncertainty of interpretation. Accompanying this trend has been increasing use of interviewing as a research methodology, especially to further investigate understanding that has already been examined through written responses (e.g., Watson and Kelly, 2003; Reading and Shaughnessy, 2004). Of particular interest to educators is whether responses to assessment items are interpreted as students intended and identification of teaching strategies that assist students to develop their understanding. Petocz and Reid (2003) provide a useful framework for analysing students' conceptions of learning and teaching statistics. This report focuses on what can be learnt by interviewing students and how this informs teaching and research.

BACKGROUND

The *Understanding of Variation* project (Reading and Reid, 2004) assessed development in understanding of variation, and identified teaching strategies that assisted the development. The focus was on tertiary students studying an introductory statistics course that treated variation as a core concept. The sequence of teaching activities was designed to give more structure to the link from students' initial 'intuitive' understanding of variation (minute papers, group discussions), to a better understanding of variation (hands on demonstrations, computer simulations, written tasks). Reasoning statistically about variation, a necessary component of making this link, requires students to explain what they are doing and why they are doing it, i.e., reason. To assess the level of reasoning about variation, hierarchies of consideration of variation were developed, based on the analysis of student responses to the various learning activities: pre-study and post-study questionnaires (Reid and Reading, 2006), minute papers (Reading and Reid, 2004; Reid and Reading, 2004) and assignments and class tests (Reid and Reading, 2005). Important research issues arise with such analysis, including whether researchers are interpreting responses correctly and whether students are responding optimally. The focus of this report is interviews, following the pre- and post-study questionnaires, designed to clarify these issues.

THE STUDY

The four question pre-study and post-study questionnaires were identical and focused on variability (Q1), comparing data sets (Q2), sampling (Q3 and Q4) and probability (Q4). Q1 asked for the meaning of variability. Q2 asked for the description, and comparison, of the timetable performance of two buses with data supplied as graphs. Q3 asked for an opinion on a statement

about the spread of children born with missing limbs in NZ, with a map divided into five regions and detail about the distribution of such births and the population spread in the regions. Q4 asked students to make, and justify, predictions about sampling from a mixture of coloured lollies. Reid and Reading (2006) reported the questions and the hierarchy used to code responses with levels of no, weak, developing or strong *consideration of variation.* Selected students were interviewed after they had completed the questionnaires to address the research questions: What extra information can be gained about students' reasoning by interviewing students? What teaching and learning activities do students perceive as assisting their development of understanding?

METHODOLOGY

The interview following the pre-study questionnaire ("pre-interview," see protocol in Figure 1) was designed to clarify what students meant by expressions in their written responses and to probe and prompt students to provide further reasoning. Explanations were checked for any change in coding level. Selection of students for pre-interviews was based on questionnaire responses that needed clarification. The interview following the post-study questionnaire ("post-interview," see protocol in Figure 2) was used to investigate students' reasoning when presented with situations of cognitive conflict based on their own responses to the pre- and post-study questionnaires. Students were asked to explain awareness of their own understanding and to identify those activities that they considered to be most effective in progressing, or hindering, the development of their understanding. Selection for post-interviews was based on questions where there had been differences in the level of the pre-study and post-study coding. Each student was interviewed for two questions. All interviews were audio-taped and conducted by a non-teaching researcher but analysed by two researchers, one teaching and one non-teaching.

For EACH question in the pre-study questionnaire:
- a. Clarify any aspect of the response that is unclear
- b. Probe for any more information that the participant may be able to add
- c. Prompt the participant into possible extension to a more in-depth response

Figure 1: Protocol for pre-interview

1. For each response that was different in level of understanding from the pre-study questionnaire:
   - a. Show the student the two responses
   - b. Ask the student why he/she responded differently the second time
   - c. Ask the student which response he/she now considers to be the better response
   - d. Ask why he/she now considers this response to be better
2. Ask the following questions:
   - a. How do you know when you understand a concept in statistics?
   - b. What aspects of the course helped you to develop these understandings?
   - c. Do you understand the concept of variation?

IF response to c is YES ask:
- d. How do you know you understand the concept of variation?
- e. What aspects of the course helped you to develop this understanding?

IF response to c is NO ask:
- d. How do you know you do not understand the concept of variation?
- e. What aspects of the course hindered your development of this understanding?

IF specific teaching strategies related to developing the concept of variation were not mentioned in the above responses then:
- f. Ask whether specific strategies have helped at all.

Figure 2: Protocol for post-interview

RESULTS FOR PRE-INTERVIEWS

Of the 32 students who completed the pre-study questionnaire six students (pseudonyms); Bron, Deb, Cassie, Adam, Bruce, and Colin were interviewed and results follow.

Clarifying

The clarifying questions generally took the form "what do you mean by……". Altogether 42 points of clarification were sought. Three of these clarifications involved some aspect of the written response that could not be interpreted by the researchers, and for each the student was able to satisfactorily explain the meaning he/she had intended. For example, Deb explained that her use of "evenly skewed" to describe the outbound bus data (Q2) was meant to describe a symmetrical distribution but she did not know the appropriate term.

For the other (39) clarification points, researchers lacked confidence in their interpretation of the students' meaning. Of these, students were unable to clarify 11 queries. Examples follow. Cassie was asked, in Q1, about her statement that "variability means differing aspects of characteristics or quality among a group" and Deb was asked to clarify her comment that "variability means differentiation." Although general conclusions can be drawn about their intentions, neither student was able to explain the intended meaning. During the interview Bruce described, in Q2, "most of the data within here, or most of the data between here and here" (pointing to values). He agreed that it was not worded well but was not able to give an alternative, leaving the interviewer assuming that only range was intended.

For the remaining 28 situations students clarified interpretation. All but three interpretations agreed with those made by the researchers. For the three non-agreeing interpretations, two instances arose in Colin's responses but neither suggested that the response should have been coded at a different level. In Q3 his observation that the southern regions "having no normalities is surprising" had been interpreted as meaning he expected abnormalities in these regions due to factors not given in the question. In fact, he clarified that if the regions have equal proportions of population then some abnormalities should be expected in each region. In Q4 Colin had stated he wanted the mean of the numbers to be five, which put no restrictions on the spread of the numbers, but he clarified that he meant "closer to five" making the spread smaller than would be suitable. The only clarification that led to a suggested change in coding (from weak to developing) was for Adam's Q1 response about variability as "amount of difference between the sets of scores," which was clarified in relation to diagrams he had drawn as meaning the distance between points on one diagram. Previously this had been interpreted by the researchers as high-low variability comparisons between the two diagrams.

Probing

The probe questions:

- directly asked students for more (e.g., "is there anything else you might like to add?"),
- implied that more was needed by repeating part of the student statement but trailing off to allow the student to continue (e.g., "you're saying there are other factors that could …"),
- brought together points made by the student and repeated them for comment,
- directed the student to a particular part of the response (e.g., "is it easier with your example?"), or
- put the onus back on the student to provide more by redirecting a question that the student asked (e.g., "that's what I'm asking you").

Probing had two purposes; as a stimulus for discussion to continue after a clarification or a prompt, and as a general probe for more information. When used as part of a clarification or prompt, probes helped to keep the student talking, especially when there was a new student-introduced idea emerging. For example, a probe in Q3 summarizing Cassie's explanation about the need for more information such as influential factors other than those given in the question, allowed her to move on to suggesting that samples from other years would also be useful. Some students gave quite detailed information on initial triggers, such as a clarifying question, but for others probing was necessary. As the specific probing was part of clarifying or prompting it was not relevant to consider any effects on the coding level. When a general probe was used, there was no more information given and hence no changes to coding levels. Some dissatisfaction with responses was expressed, such as Deb who admitted that in Q3 she had taken the wrong approach and Bruce who said he could have given a better answer in Q2, despite claiming that the question

was poorly worded. However, the general probe was usually given after both clarifying and prompting that had already allowed the student to share relevant ideas.

Prompting

The prompt questions:

- questioned the quality or basis of given responses (e.g., "is that important?" , "…. was based on the fact that ….?", "why did you ….?"),
- suggested alternatives to consider (e.g., "do you think you could get none?"),
- asked for a reason for something (e.g., "…because…."),
- suggested a point as a way forward in explaining the response (e.g., give alternate representations, summarize information, draw a picture), or
- asked a specific question leading the student to consider a point involving a deeper level of thinking.

Some students reacted favourably to prompting. Minimal prompting was needed to assist male students to elaborate on their written responses, but the three female students did not respond so well. Prompting elicited little more from Bron whose responses were already developing or strong. Deb needed a lot of prompting to encourage her to advance her responses, and Cassie generally did not manage to run with leads she was given, often just agreeing with a prompt rather than being able to use it as a stimulus to develop more discussion.

The most important prompts to consider are those that allowed the students to produce changes in the level of response. Seven prompts produced increases in the level but two resulted in a weaker level. The prompting appears to have been most effective for Q3; Deb (weak to developing), Bruce (developing to strong), Colin (weak to strong) and Cassie (weak to strong). One student showed improvement in each of the other questions; Deb (Q1, weak to developing), Adam (Q2 developing to strong) and Bruce (Q4, weak to developing). The two occasions when prompting decreased the coding, from developing to weak, were Q2 for Cassie and Q4 for Colin.

RESULTS FOR POST-INTERVIEWS

Four students (pseudonyms); Anne (Q1, Q4), Ellie (Q2, Q4), Adam (Q2, Q3), and Bruce (Q1, Q3), were interviewed after the post-study questionnaire. These students were chosen from those who had a post-study response coded at a different level to the corresponding pre-study one.

Cognitive Conflict

Students found it difficult to make a clear decision when presented with a choice between their own pre- and post-study responses to a question, often being influenced by the fact that most post-study responses included evidence of the courses' teaching and learning activities. Students often used standard statistical terms (e.g., standard deviation) and referred to numerical and graphical summaries (e.g., a sampling distribution) that had been presented in class. However, evidence that the student understood these inclusions was lacking. In explaining his preference for his post-study Q1 response, which displayed a weaker level of consideration, Bruce said that he "just tried to look at it from a more - I guess - mathematical, statistical view" and provide a numerical summary. There were two other instances where students preferred the weaker response. Ellie's justification for her choice in Q4 was that she had misinterpreted the post-study question. Anne chose the weaker response (pre–study) in the first part of Q4, because it was "really vague." Further discussion led her to comment that a combination of answers that "… doesn't contradict itself on one page, [is] always good." She was uncertain about her responses and worried by apparent contradictions. In two cases, students developed a third response in preference to either of their previous responses. Ellie decided to revise her response to the last part of Q4 after her misinterpretation of the question became apparent. Anne, after prompting, drew on a practical example to help formulate a better response to Q1.

Teaching and Learning Activities that Impacted on Statistical Reasoning

All four students believed they understood a statistical concept if they could apply that concept in a variety of contexts. Each of the teaching and learning activities was deemed, by at

least one student, to have helped develop that understanding. These included well-structured lectures; step-by-step instructions for applying methods; the use of minute papers to identify knowledge gaps for use in self-directed learning; computing assignments that brought together theory and practice; working on tutorial questions prior to the class to maximize the benefit of direct interaction with teaching staff; and instantaneous feedback in a small class setting.

When asked if they understood the concept of variation, Adam, Bruce and Ellie all agreed, although Adam and Bruce could not explain how they knew that they understood. Ellie explained by defining the concept in a similar way as the approach to Q1 in the questionnaire. When asked Anne replied, "probably not" and explained that "there hasn't been a definition given for it." The students found it difficult to identify specific teaching and learning activities that had helped develop their understanding of variation. Perhaps they felt that they had already discussed them with reference to their understanding of general statistical concepts, or they may have struggled to answer since "variation" is a more abstract concept. Ellie and Bruce mentioned statistical concepts such as sampling distributions and the link between sample size and the standard error but only Ellie cited a particular activity – a minute paper that had considered an intuitive analysis of variance - as useful in developing her consideration of variation. A common theme that did emerge was the use of graphics and visuals as an important tool in helping students develop a better understanding of statistical concepts in general, and of variation in particular.

DISCUSSION

A wealth of information about the way that individual students express their reasoning about variation was gained by interviewing students using clarifying, probing, prompting, and cognitive conflict situations. Clarifications indicated that generally researcher's initial interpretations of written responses were consistent with those students intended but the value of interviewing is demonstrated better by the probing and prompting. Probes helped students to continue their thread of discussion and better articulate their explanations, although the general probe as a final query did not produce any relevant information. However, probing is not sufficient to improve the quality of a student response. There needs to be some form of prompting and even then some students do not necessarily improve. A possible explanation of the higher number of improved quality responses for Q3 was that the wording of the question was more general than the other questions and with prompting the students were better able to unpack the question. The two occurrences of reduction in levels, after prompting, serves to demonstrate that there may be some instances when students are able to provide standard statistical expressions but not able to explain what they mean.

Despite prompting being more readily utilized by the males than the females, to expand on responses, all but one student was assisted to increase the level of at least one response. Those prompts that built on information or ideas supplied by the student were more successful that those that suggested a way forward that was not linked to student-initiated ideas. Students found it difficult to make choices in cognitive conflict situations and some preferred the post-study responses using concepts and terms from the course, even though they did not necessarily reflect understanding. The listener needs to be sensitive to the thinking path of the student and to structure prompting and cognitive conflict situations accordingly.

Student explanations of self-awareness of conceptual understanding provided useful insights into their conception of learning. With reference to Petocz and Reid's (2003) conceptual framework for statistics learning, two students viewed their learning as "applying" and the other two as "linking." None of them was able to articulate the preferred view of the value of statistical learning in "learning about areas outside statistics" or to "support changing one's views." Students attributed the development of their understanding to a wide range of teaching and learning activities and generally explained how each contributed. However, they showed poor conceptions, according to Petocz and Reid's (2003) conceptual framework for statistics teaching, as simply "providing materials, motivation, structure," or "explaining material and helping with student work." Only one student conceived teaching as "linking statistical concepts and guiding learning." Preferred conceptions of teaching would be as "anticipating students needs" or "being a catalyst for open mindedness." Although no particular strategy was favoured for learning, the desire for visual aids and graphical representations was a recurring theme.

This study was exploratory in nature, limited by the small number of interviews and the use of audio-tape, rather than video-tape, not capturing information about actions when students pointed to parts of their responses. Despite its limitations, this study has demonstrated the usefulness of probing, prompting and cognitive conflict to encourage students to talk about their reasoning about variation. It has also shown that students' awareness of the nature of their own understanding, and the teaching they experienced is not well developed conceptually. Strategies are needed to assist students to change their view of teaching from a simple provision of essentials to a focus on student learning.

IMPLICATIONS FOR TEACHING AND RESEARCH

Student explanations of their unique expressions and additional reasoning in prompted and cognitive conflict situations were shown to provide a deeper insight into how individual students reasoned about variation. Such processes could be used to understand better how students reason, and highlight the importance of listening to students. As students identified a variety of teaching strategies that benefited their developing reasoning, educators need to plan a range of activities, including opportunities for students to talk about their work. Facilitation by prompts and cognitive conflict situations would encourage development of their reasoning. Researchers are challenged to determine whether similar techniques are effective in obtaining deeper insights into student reasoning about other statistical concepts. Finally, the extra insight into reasoning provided during clarification and in response to prompts and probes highlights the value of interviewing, a time consuming but rewarding data collection process for researchers. To understand and support students' reasoning about variation we need to listen to the students.

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