# FORMING SMALL CLASS GROUPS USING MULTIDIMENSIONAL SCALING

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*In college courses that use group work to aid learning and evaluation, class groups are often selected randomly or by allowing students to organize groups themselves. This article describes how to control some aspect of the group structure, such as increasing schedule compatibility within groups, by forming the groups using multidimensional scaling. Applying this method in an undergraduate statistics course has resulted in groups that have been more homogeneous with respect to student schedules than groups selected randomly. For example, correlations between student schedules increased from a mean of 0.29 before grouping to a within-group mean of 0.50. Further, the exercise motivates class discussion of a number of statistical concepts, including surveys, association measures, multidimensional scaling, and statistical graphics.*

## INTRODUCTION

I have found the use of group work in my undergraduate statistics course to be a useful method for improving student learning by raising student interest and increasing class participation. As well as working together during class time, students work extensively together in groups outside of class on homework assignments and projects. In managing groups of 3-5 students in a class of approximately 60, I have experimented with various unsatisfactory methods for selecting the groups. For example, randomly selecting groups has lead to student complaints about difficulties meeting as a group outside of class time due to incompatible schedules. On the other hand, allowing students to self-select groups has tended to produce groups of friends in which there is very little diversity (gender, age, and ethnicity, as well as academic ability). To address this, I have developed a method that uses multidimensional scaling (MDS) to select groups whose members have mostly similar schedules without compromising group diversity.

The concept of cooperative learning involves groups of students working together as a team to solve a problem or complete an assignment (see Garfield (1993); and Keeler and Steinhorst (1995) for examples in the field of statistics). Johnson, Johnson, and Smith (1991) showed that when students work together, they often accomplish more, and at a higher level, than they could individually. Garfield (1993, par. 8) cited published research that suggests that "the use of small group learning activities leads to better group productivity, improved attitudes, and sometimes, increased achievement." In selecting cooperative learning groups, Garfield (1993, par. 18) noted that "the instructor may allow students to self-select groups or groups may be formed by the instructor to be either homogeneous or heterogeneous on particular characteristics (e.g., grouping together all students who received A's on the last quiz, or mixing students with different majors)." The remainder of this article describes how to use MDS to select groups to be homogeneous on student schedules. The method also enables inclusion of further selection criteria, such as ensuring groups have at least one member with a particular skill. The method is described in sufficient detail that it can be applied to any course with cooperative learning groups, and can be adapted to work with characteristics other than student schedules. A more comprehensive version of the article which develops these ideas further is in Pardoe (2004).

The next section briefly describes MDS and how it can be applied to group students with similar schedules. The following section presents typical results and evaluates how well the method has worked in practice. Implementing the method also provides valuable opportunities for covering a number of statistical concepts in class, including surveys, association measures, multidimensional scaling, and statistical graphics. The final section contains concluding remarks.

## MULTIDIMENSIONAL SCALING FOR SELECTING GROUPS

MDS allows a set of objects to be displayed in low-dimensional space (often 2D) to reflect similarities between the objects (see Kruskal and Wish, 1978, for an overview). Physical distances between the objects on a 2D MDS map are optimized to correspond with measured object similarities. The input to the procedure, a matrix representing pairwise similarities between the objects, can be obtained directly by eliciting explicit object similarities from survey respondents, or indirectly by calculating an association measure for the objects based on the values of object covariates. For this application, the goal is to identify groups of students (the objects in question) with similar schedules, so I use the latter approach using calculated associations between students based on their schedules.

A metric MDS analysis (Torgerson, 1958) uses interval or ratio scaled similarity measures, and an eigenvalue decomposition of (a transformation of) the matrix of similarity measures provides the map coordinate locations of each object. Similar to a principal components analysis of a covariance matrix, the first eigenvector represents the first coordinate axis (with the largest variance in distances), the second eigenvector represents the second coordinate axis, and so on. When, as is often the case, the first two eigenvalues account for the bulk of the distance variance, a 2D solution can map the objects effectively. When the similarities are ordinal measures, a nonmetric MDS analysis (Shepard, 1962) places the objects on a map to preserve a monotonic relationship between observed similarities and map distances. A measure of badness of fit, or *stress*, summarizes how far a proposed solution is from the desired monotonic relationship, and a numerical algorithm iteratively adjusts the object coordinates to minimize the stress. Metric and nonmetric MDS analyses thus operate on different data scales using different approaches to produce maps that effectively represent similarities between objects.

Because it is possible to calculate a ratio-scaled measure of association between student schedules, a metric MDS approach is appropriate in this application. The process begins with collecting schedule information using, for example, a pen and paper survey, e-mail with an attached spreadsheet to enter data, or an online survey. In a statistics class there is clearly an opportunity here to engage the class in a discussion of data collection techniques and surveying. For the undergraduate statistics course that I teach, the students complete an online survey before the second class of the term. I use *WebSurveyor* software (www.websurveyor.com) which makes this process particularly easy, resulting in a spreadsheet of student data. I ask the students to "select the time periods when you absolutely cannot meet with your group because you are in class, work, or other scheduled activity for most or all of the period during the term." I use two-hour time periods between 8:00 a.m. and 10:00 p.m. for all days including weekends – this seems to provide a reasonable compromise between survey burden and information quality. The resulting spreadsheet records a row of zeros and ones for each student showing when they are available/unavailable to meet for group-work outside of class time. This spreadsheet also records a unique identification number for each student, and binary indicators for whether a student self-selects as "good with computers" and "good at analytical thinking."

Administering the survey and explaining its purpose provide opportunities for discussing statistical concepts in class, for example how to measure the similarity of one student's schedule with another. This naturally leads to the notion of an association measure, in this case for data that can be summarized in a two-by-two contingency table: the two row categories are the counts of the available/unavailable time periods for one student while the column categories do the same for another student. For example, a typical table for a pair of students might contain the following: both students available, 27 periods; both unavailable, 6 periods; student A available but student B unavailable, 9 periods; B available but A unavailable, 7 periods. One way to measure association in this table is to look at a scaled difference between concordant pairs of time periods (where students match and are both available or unavailable) and discordant pairs (where students do not match) – Kendall's $\tau_b$ (Kendall, 1945) is one such quantity. For two-by-two tables, Kendall's $\tau_b$ is algebraically the same as the usual (Pearson product-moment) correlation between each student's availabilities, and so this might usefully lead into a discussion of bivariate correlation in more general situations.

Returning to the group formation task, statistical software can then be used to perform an MDS analysis. I use *SAS* software, although many other common software packages could be used. The *SAS* code used in this article is available at http://lcb1.uoregon.edu/ipardoe/research.htm. Output from the MDS procedure includes a 2D map where physical map distances between the students correspond to the similarities between their schedules. Figure 1 provides an example of such a map for 58 students in an undergraduate statistics course that I taught recently.

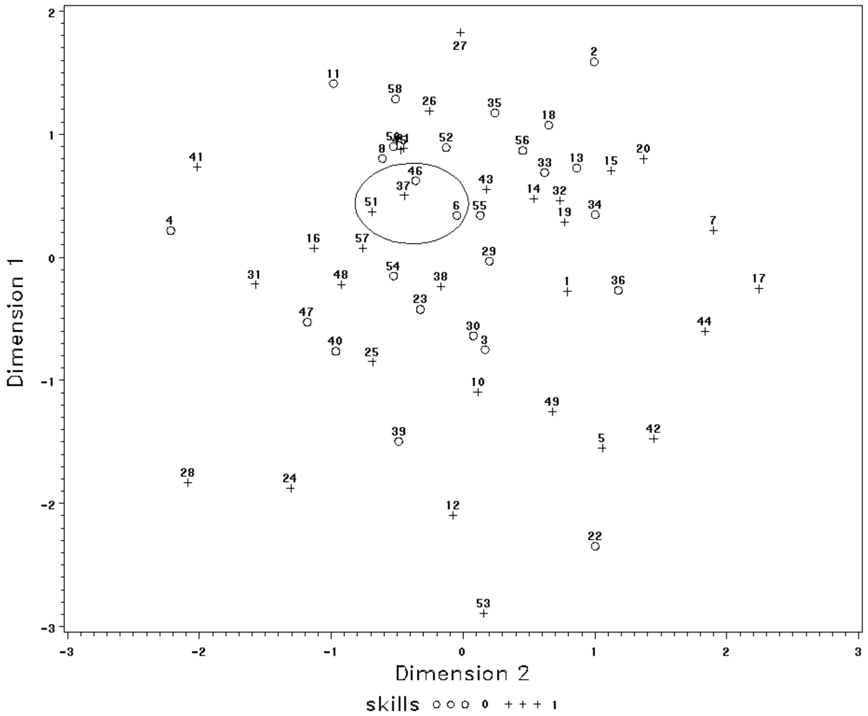


Figure 1: Perceptual map showing students labeled by identification number and marked according to whether they have computing/analytical skills (circles: no, crosses: yes).

All that remains is to use this map to determine the groups. I have found that it is usually sufficient to print the map out, and then, by eye, delineate boundaries between groups such that there are four or five students in each group, and each group has at least one member claiming to have computing or analytical skills. A more formal clustering technique might be used at this point – providing another opportunity to introduce a statistical concept into class – but I have found little need to venture beyond informally eyeballing the map.

EVALUATION OF RESULTS

Since the map in Figure 1 represents similarities between students' schedules (to the degree that the MDS analysis has been successful at representing high-dimensional data in 2D space), using the map to select groups ought to produce more homogeneous groups than selecting groups randomly. For example, I used the map to place students 6, 37, 46, and 51 together (see Figure 1) – their 6 within-group correlations ranged between 0.5 and 0.8, whereas the 1653 correlations across the whole class went as low as –0.5. To evaluate the success of the group selection process, I considered this question in more detail. One way to do this (which also provides a useful lead-in to discussing graphical summaries in class) is to compare a histogram of schedule correlations for all pairs of students with a histogram of correlations within the final selected groups. For the groups selected in the example above, the distribution of correlations shifts to the right, so within-group correlations tend to be higher (averaging 0.50) than correlations across the class as a whole (averaging 0.29). There were relatively more within-group correlations at the high-end of the scale, and so in the most homogeneous groups (comprised of very tightly clustered students in Figure 1), schedules matched remarkably well. However, while no within-group correlations were below –0.1, there were some students with schedules that matched few others in the class, making it difficult to place them in a homogeneous group. For example, it would have been difficult to place student 28 in a homogeneous group – this student's optimal group would have had a minimum correlation of 0.1, and they actually ended up in the group with the lowest within-group correlation of –0.1.

This last issue motivates another discussion question for potential use in class – why not just try to find the optimal group for each student? Apart from being extremely time consuming (particularly as the class size grows), what is optimal for one student may not be optimal for another – for example, placing student 28 in their optimal group would have changed the group membership of a number of other groups and greatly reduced their homogeneity.

## CONCLUSION

Randomly assigning students to small groups in college courses, or allowing students to self-select groups, can impact operational aspects, such as how well groups are able to schedule times to meet, as well as personal characteristics, such as student diversity within groups. To avoid such problems, the process described in this article uses multidimensional scaling for selecting groups to increase scheduling homogeneity within groups without reducing diversity. The approach has worked successfully in practice and student feedback has been very favorable. In theory, within group diversity remains as high as for random group selection, although to the extent that "similar students" have similar schedules, this may not strictly hold in practice. The approach can also take into account additional student characteristics, such as making sure each group contains at least one student with a particular skill. The method as described uses Pearson correlations (equivalent to Kendall's $\tau_b$ in this case) to calculate student schedule similarities, but it is possible to adapt the method to use alternative association measures, for example if it was desired to consider matches on times when students can meet as more important than matches on times when students cannot meet. An added benefit, when applied to statistics courses, is that the approach offers many opportunities for exploring statistical concepts in class.

I have focused on using the approach to enhance within-group schedules, but the approach is general enough to consider other characteristics too, such as student grades or majors (as mentioned by Garfield, 1993), or skills. For example, suppose it is desired to form groups in which students have a wide variety of skills and competencies (such as analytical, writing, organizing, public speaking, leadership, computing, detail-oriented, good with "big-picture," etc.). Scores (from 1 to 10, say) on each of these skills can be obtained via a student survey, and then a metric MDS analysis can be based on pairwise correlations between student scores. In this case, the goal would be to group students with negative correlations so that each group has a good mix of skills. Thus, students with very different skills could benefit from being placed in the same group, while students with similar skills could benefit from being placed in different groups.

The general approach involves some start-up costs. These include: collecting the relevant information from students (for example, scheduling information provided through an online survey); analyzing the data (for example, using PROC MDS in *SAS* to create a map of students in which those close together have similar schedules and those far apart have very different schedules); using the map to select groups (for example, informally clustering the students by eye). Once the survey and *SAS* code have been written however, it is easy to implement the approach for any class where homogeneous groups need to be formed.

## REFERENCES

Garfield, J. (1993). Teaching statistics using small-group cooperative learning. *Journal of Statistics Education*, 1(1), http://www.amstat.org/publications/jse/v1n1/garfield.html.

Johnson, D. W., Johnson, R., and Smith, K. (1991). Cooperative learning: Increasing college faculty instructional productivity. ASHE-ERIC Higher Education Report. 20(4). Washington, DC: The George Washington University.

Keeler, C. M. and Steinhorst, R. K. (1995). Using small groups to promote active learning in the introductory statistics course: A report from the field. *Journal of Statistics Education*, 3(2), www.amstat.org/publications/jse/v3n2/keeler.html.

Kendall, M. G. (1945). The treatment of ties in rank problems. *Biometrika*, 33, 239-251.

Kruskal, J. B. and Wish, M. (1978). Multidimensional scaling. *Sage University Paper on Quantitative Applications in the Social Sciences*, Newbury Park: Sage Publications.

Pardoe, I. (2004). Multidimensional scaling for selecting small groups in college courses. *The American Statistician*, 58, 317-321.

Shepard, R. N. (1962). The analysis of proximities: Multidimensional scaling with an unknown distance function. *Psychometrika*, 27, 125-140; 219-246.

Torgerson, W. S. (1958). *Theory and Methods of Scaling*. New York: Wiley.