SEMIOTICS FUNCTION AND LEARNING OF ARITHMETICS MEAN

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This paper continues earlier studies about the teaching and learning of the arithmetic average and it is part of a broader research in progress at Santiago of Compostela University (Spain). We have analyzed a sample of six teaching manuals (textbooks) used for teaching mathematics at high schools in Salvador, Bahia. The study is based on theoretical ideas by Godino and Batanero (1994; 1998) and Godino and Recio (1997) who propose a semiotics perspective based on the functions of signs by Hjelmslev (1943), later known as""semiotic function"(Eco, 1979).

INTRODUCTION

The effort of the mathematics teaching community on the use of semiotics is shown through works such as the ones done by Ernest (1993) and Vile and Leman, 1996), as well as the studies made by Bauersfeld and his collaborators (Cobb and Bauersfeld, 1995) which emphasized the negotiation of meanings as the main points in mathematics teaching.

In this paper the meaning of the arithmetic average has been analyzed in some mathematics textbooks used in the City of Salvador, in Bahia. The general aim is contribute to improve the teaching and learning of statistics and, in particular, the conceptualization of the arithmetic average. Specifically, we intend (1) to start a theoretical debate in education about what is involved in learning statistics as a part of mathematics; (2) to assess the institutional meaning of a mathematical object and compare with students' interpretation and decisions about the problems associated to this content (arithmetic average). To attain these goals we focused on the theoretical model developed by Godino and Batanero (1994); Godino and Recio (1997), since this model defines clearly the difference between institutional and personal meaning of a mathematical object. Even when our interest is the institutionalized mathematical knowledge, we cannot, however, forget students and their individual capacity of development.

PREVIOUS RESEARCH

Many researchers have focused on statistics education; among them we highlight the studies by Watson and Moritz (2000) which developed a hierarchic model of cognitive understanding of the arithmetic average, taking in account the importance of the context in which it is applied. Gal, Rothschild and Wagner (1990) observed that students rarely use the mathematical average spontaneously when comparing data sets. Mokros and Russell (1995) in their investigation concluded that students develop some notion of average as a representative value of a data set, without understanding how to apply the concept. Finally, a more comprehensive research is developed by Cobo (2003) who carried out a theoretical and experimental study on the meaning and understanding of position measures at Spanish high schools. She analyzed the types of problems, representations, procedures, definitions, properties and arguments related to averages, both from institutional and personal points of view. She carried out an epistemic analysis of 21 textbooks, which enabled her to determine the institutional meaning of reference for position measures. She finally built a questionnaire to assess students' difficulties when facing this content.

METHOD

Our methodological procedure to analyze 6 textbooks follows the model developed by Cobo (2003), for each of the books we defined 3 units of analysis in the content and arithmetic average: Concepts and Definitions; Notations and Symbols; Exercises. The procedures involved detailed readings of the textbooks, comparisons with institutional meaning of reference and building of comparative charts to highlight the characteristics of the analyzed texts.

ANALYSYS AND RESULTS

1-Conceptualization and Definition of Average. According to Goodchild (1988) the arithmetic average is a measure of central position for data distribution. Most authors define arithmetic average as: a) "*the arithmetic average is a characteristic value of a set of data*" or b) "*the arithmetic average is a value of central position.*"

We notice that the language presented in the majority of textbooks is very similar. That is, the terms and expressions, in a general way, portray the concept of the arithmetic average, as being a central tendency measure that represents through a unique number, the characteristic or central value for a data set. Additionally, some authors present the algebraic concept for the arithmetic average by constructing a conceptual definition for the average through a formula given by algebraic notation. These definitions, of course, can lead students to recognize only part of characteristics related to the arithmetic average, leaving aside, however, other important properties, such as being the best estimator for an unknown value from repeated measures. Below, we present an example of implicit conceptualization of the arithmetic average from one of the textbooks analyzed.

Book	1. Concept and definition of the arithmetic average
1.4	…Now we will establish some representative values for these data (numbers), that is, a way to summarize values attributed to a quantitative variable. And for this purpose, it is necessary to establish an average or central value… Gelson Iezzi, Osvaldo Dolce, David Degenszajn e Roberto Pergio, 2005, p. 422-423.

2-Notation and symbols. In the majority of the books analyzed, the authors introduce formulas and notations to facilitate the solution of problems or compute the arithmetic average via a simple numerical operation- After that, they present the compositions of these formulas through the conjunction of some usual mathematical symbols related to the previously used expression. For example, they introduce the notations and representations: (v.a) variate, (xi) element, ($\overline{X}$) arithmetic mean, ($\sum$) addition, (n) sample size, among other usual mathematical and statistical symbols. We observed in our analysis that the authors usually assume that students know what is represented by each of these expressions,

Therefore, there is an implicit agreement of authors that students are capable of developing with some ability the operations involved in the use of statistical notations, relating them to numerical values represented by each symbol so that later they will be able to use it as a variable. To follow, we present an example where algebraic notations and symbols are used to calculate the arithmetic average.

Book	2.Notation , symbolic representation of"computation of averages
2.1	The arithmetic average *(X)* of the values $X_1, X_2, X_3, \ldots, X_n$ is the quotient between the addition of these values and the total number "n" : $\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$ From *Fundamental Math – A New Approach*. By Giovanni, Jose. Bonjorno, Jose Roberto e Giovanni Jr., Jose Ruy – 2005, p. 466.

3-Exercises. At the third stage of our analyses we have analyzed the examples and exercises in the textbooks that had been previously selected, describing their characteristics, concepts and relations needed to solve them. In the majority of books analyzed, the authors include exercises that only guide students to perform systematic calculations, or either to apply simple mathematical operations to obtain the arithmetic average. This strategy combines with the reduction of students' difficulties to apply Mathematical rules. We notice the authors' concern for presenting new resources in the elaborations of exercises as a way to improve the interpretation of the concept of arithmetic average. In this sense they present examples with graphs and tables that have the aim of allowing understanding of data and their relationship with averages. It is a concern of the authors to demonstrate the utility of the arithmetic average as another resource in problem solving. In the example below, the idea of monthly estimate of births is presented.

<table>
<tr><td>Book</td><td>3. Exercise</td></tr>
<tr><td>3.1</td><td>In the year 2000, the number of births, per month, in a maternity hospital was:
<table>
<tr><td>Month:</td><td>J</td><td>F</td><td>M</td><td>A</td><td>M</td><td>J</td><td>J</td><td>A</td><td>S</td><td>O</td><td>N</td><td>D</td></tr>
<tr><td>Birth :</td><td>38</td><td>25</td><td>42</td><td>30</td><td>29</td><td>47</td><td>18</td><td>36</td><td>38</td><td>43</td><td>49</td><td>37</td></tr>
</table>
a) Compute the monthly average of births.
b) In which months the birth number was above the average?
Mathematical Context and Applications. Dante, Luiz Roberto – 2004 p. 228.</td></tr>
</table>

CONCLUSIONS

In the introduction we directed our attentions to the ontologic-semiotic model in the perspective of the notion of meaning as a central point of teaching mathematics (Cobb and Bauersfeld, 1995). In our research this model was associated with the notion of "institutional and personal meaning for a mathematical object" (Godino and Batanero, 1994), For example, in the "arithmetic average," there is a personal meaning for each subject that might be shared or not with the institutional meaning of the concept. Our analysis was aimed to identify the elements that we have studied as possible conflicts between the personal and institutional meanings - the book and its didactic relation. According to Godino (2002), these semiotic conflicts must help us understand the students' difficulties and limitations in understanding mathematics. From our conclusions in this study we can produce the first syntheses for each unit of analysis:

In the conceptualization and definition of the arithmetic average, we observed that authors, in general, conceptualize the arithmetic average with generic situations, but at no point justify the particular use of the arithmetic average that is reasonable in some stages of data analysis. Moreover, many authors forget to focus on these ideas: the arithmetic average as a measure that expresses balance, satisfaction and esteem amongst other qualities. In this regard, we believe that a careful elaboration is a must since the users of these books are the secondary school students who are starting their first contact with these values in Mathematics.

Regarding notations and symbols, the authors have introduced demonstrations and presented different notations which the literal part of the formula of the arithmetic average is not clarified. This might have a negative effect on students' exact use of formulas since, in many cases, they haven't still acquired the knowledge of the utility of many symbols and notations in the context of statistics, and they only understand simple notations to calculate the average. This difficulty in the elaboration and the definition is perhaps one of the main concerns today as regarding arithmetic average. A better elaboration requires the definition of only standard formulas, where the elements that compose each notation will be dealt with a general rule.

Finally, the analysis of the exercises shows that there is a concern of making the students get the summary basic directions when getting quantitative value between the data that define the average. There is no concern for the relationship between the nature of the phenomena and the implication given by the calculations since the only interest is applying a mathematical rule to solve the exercise, without taking into account the nature of the phenomenon or variable. This inexistence of contextualization implies in lack of applications of the arithmetic average properties, however, some books change when using graphs and table representations. This initiative can improve the interpretation and application of the arithmetic average.

In summary and taking into account our theoretical framework, it is possible to legalize the conditions for a better understanding of the ways in which students attribute meanings to the terms, mathematical symbols, concepts and procedures for a formative perspective (Godino and Batanero, 1994).

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