AN INSTRUMENTAL OBSTACLE EXAMPLE

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The aim of this paper is to present the concept of an 'instrumental' obstacle. In French agricultural education, the spreadsheet is often used as a tool or "artefact" in statistics teaching. Some obstacles to learning appear due to the use of this instrument. Difficulties appear during the learning of analysis of variance by students, who are not trained mathematicians. The concept of average however, which might have been regarded as unproblematic, caused surprising difficulties during one step in the algorithm for analysis of variance. The notion of 'instrumental' obstacle seems to be pertinent in order to analyse this phenomenon. This kind of obstacle is different from those presented by the internal constraints of the artefact. This study confirms that students have yet some problems with the notion of average, but that with spreadsheet use they become aware of this difficulty.

THEORETICAL FRAMEWORK

With the ready availability of digital technology, the calculation of average is available to almost everyone. However, availability may not be enough. Although technology may provide the calculating power, such resources will not be exploited, or at least not exploited correctly, without a conceptual basis for appreciating the meaning of average itself. Thus we must consider the role of the artefact and instrumented activity (Trouche, 2004) as the main issue of this proposition. We decided to study how students developed tools to solve the difficulty of average by exploring the socio cultural dimension of students' behaviour as they engaged with the notion of weighted mean.

It seems that in the use of the spreadsheet in mathematics lessons, there is an interaction between learning how to use the spreadsheet and building statistical knowledge. In the process obstacles (Brousseau, 1997) appear for students, even those who have experience in statistical techniques and spreadsheet use. [An obstacle is previous mathematical knowledge, badly applied or applied outside of its domain of validity.…This type of obstacle seems to be unavoidable in the construction of mathematical knowledge.]

Brousseau classifies obstacles into three types: ontogenetic obstacle, epistemological obstacle and cultural and didactical obstacle.

In this paper, we combine *cultural* (as well as socio-cultural) and *didactical* obstacle notions with the notion of *constraints of orders* as developed by Trouche. We mean artifact (or tool) as an object available as support for human activity in reference with the 'instrumental approach' (Rabardel, 1995). Trouche himself classifies *artefact constraints* into three categories: internal constraints, ergonomics constraints and constraints of orders linked with form and existence of orders available in the artefact.

Indeed, in instrumented situations that integrate artefacts and in particular computers, cultural, socio-cultural or didactical obstacles may be reinforced by their own constraints. This article tries to link the notions of didactical obstacle and constraints of orders. Through an illustrative example we propose to name such obstacle 'instrumental' obstacle.

CONTEXT IN WHICH DIDACTICAL PHENOMENA APPEAR

Teacher teach the plant biotechnology option within the Professional Degree. The course is 18 hours long. The students have a H.N.D. level (school leaving certificate + 2 years of higher education). For two years they have been required to apply conformance and adjustment tests and also analysis of variance techniques. Lessons took place either in a traditional classroom or in a computer room.

The proposed strategy for statistics teaching rests on three hypotheses:

- The importance of 'reflection speech' for students' learning (Chiocca, 1995)
- Students perform the task more successfully if they have some theoretical knowledge and if they use trial and error strategy (Chiocca, 2002)

- The use of spreadsheet for calculation prior to use of analysis tools within the software makes the control of results by the user easier

Teaching design includes instrumented situations to foster the appearance of obstacles about the notion of average. One of the main levers is the didactical contract. Our proposal is different from situations that tend to modify some of the constraints of the software in order to encourage certain types of student behavior. Here, all the possibilities offered by the spreadsheet are left open to the students. Indeed, within the interaction: use of spreadsheet/building of mathematical knowledge, obstacles appear and these are the precise object of our studies.

INFLUENCE OF SPREADSHEET USE ON THE BUILDING OF MATHEMATICAL KNOWLEDGE

The lesson considered here, expands the algorithm of analysis of variance by the resolution of the following task, suggested by Professor L Genzbittel (Professor Genzbittel is an ‘ordinary’ mathematical teacher in the sense that his teaching is not designed with a didactical researcher.) We will compare 4 varieties of potatoes in a given area. Each variety has the following output from 36 regions, each with an area of 250m², cultivated under comparable conditions. Varieties are distributed in random way across the regions.

Variety1	*14.5*	*13.7*	*15.8*	*17.2*	*12.5*	*13.9*	*14.8*	*18.5*	*12.9*		
Variety 2	*12.5*	*13.2*	*14.7*	*11.8*	*12.7*	*18.2*	*14.2*	*13.3*	*11.7*	*10.2*	*9.8*
Variety 3	*9.5*	*12.3*	*10.7*	*9.8*	*13.1*	*10.7*	*12.2*	*13.5*	*10.1*		
Variety 4	*13.2*	*12.7*	*11.6*	*10.3*	*14*	*11.8*	*10.1*				

Didactical Obstacle

The various techniques for calculating an average of data presented in extenso, grouped or in classes, are not explicitly explained by the official instructions in French curricula.

Socio-cultural Obstacle

The first calculation of average taught at school is to sum all data, then, divide the sum by the number of data. (‘Simple’ arithmetic mean). Sometimes, the calculation of the average is a little more complicated because of coefficients as in the case of a weighted mean. Most students only know about the ‘simple’ arithmetic mean. It is as if their first learning of average at elementary school overshadows later experiences of average (for example, in geometrical or physical frameworks).

Mathematics Obstacle

The obstacle around the concept of average has already been located. Indeed, it is shown (Mevarech, 1983) that ‘for the calculation of average, pupils (aged 8-14) use calculation rules, that apply to groups, in particular closing rules. 65% of pupils draw up the average of averages without taking into account the fact that samples are not of the same size.’

Gattuso and Mary (1996), and Girard (1998) show that for some pupils and students, the concept of average was not available in the form: the average is the common value if all the values are identical.

Constraints of Orders of the Spreadsheet

The spreadsheet contains an AVERAGE function that computes the arithmetic ‘simple’ mean of the values indicated (as a vector in a column or a matrix spread across rows and columns). All students know how to use this feature.

The SUMPROD function permits one to calculate weighted means (immediate for pupils, students and media). This function carries out products of matrices set out in columns of the spreadsheet. However, these students have not studied this function before.

Thus, when they have to calculate the general average of the data, they have at their disposal only one technique: use of the AVERAGE function suggested by the spreadsheet. They have a choice between:

- Indicate the region on the spreadsheet containing all the values of the 4 samples of the 4 varieties of potatoes,

- Or, take advantage, from the fact that the averages of the samples were already calculated, to deduce the general average of the averages of each sample. This approach was strongly suggested by the different steps of the algorithm such as we chose to present it. (But the students would then need to take into account the fact that the sizes of the samples are not all the same).

In other words: either, they calculate the sum of all x_i then divide it by the number of x_i, easily accessible using the AVERAGE function, or they calculate $\frac{1}{n}\sum_{i=1}^{n} n_i \bar{x}_i$, non accessible by the AVERAGE function.

Use of Spreadsheet Allows Awareness of a Difficulty

The didactic situation in the analysis of variance acts like a lure for the activity concerning the average. It is difficult for students to understand analysis of variance. It is perhaps the first time they have been asked about the comparison of more than two means. So, the students focus their attention on the analysis of variance without realising that they might have a problem when they calculate the general average of the data. This calculation is thus regarded by most of them as common and it is carried out in a quasi-automatic way.

When the students calculate the general average of the data, two results appear in class. Then, the following exchanges take place between the teacher and the students as the script of the video shows (two mobile cameras, one focused on students one focused on the teacher):

Teacher: we will denote the general average by $\bar{x}$ *.*
Student: How can we calculate the general average?
T: so how does one calculate the general average?
S : ....
T: there are 2 methods: either one takes all the data, or one makes the average of the sub-groups. Did you find the same thing?
S1: 12,8
S2: 12,83
T: Do we find the same result when we calculate the average of the sub-groups and when we calculate the average of all the data?
S: no, not exactly: 12.83 and 12.8
T: without making a round-off
S: no it is not the same

Some students used the AVERAGE function on the averages of the samples without taking into account the different sizes from the samples. Inconsistencies with other results of students who applied the AVERAGE function on all the data allows the emergence of an obstacle. Thus, the constraints of orders of the artefact linked with the didactical obstacle related to the concept of average allows the emergence of a socio cultural obstacle on the elementary technique concerning the calculation of average. At this point, the researcher advices the teacher to take advantage of the planed discrepancy of results to introduce his or her ‘reflection speech’.

The phenomenon of awareness by some students that they have badly applied knowledge about average hardly arises in ordinary lessons.

THE NOTION OF ‘INSTRUMENTAL OBSTACLE’

The phenomenon described above can be stated within the theoretical framework of the instrumental approach in the following way:

In computerized situation, aimed simultaneously at the learning of statistics and the use of the spreadsheet, one particular artefact use scheme (Vergnaud, 1991) introduces an obstacle for some students to realize the task suggested: the computing of a weighted mean screened by the variance analysis.

In the situation of instrumented activity described, we can observe the use of a *common scheme* (computing mean = use of AVERAGE function) by some students. In the course of execution, the students are confronted with inconsistent numerical results, and the common scheme allows a new significance to arise: when the samples are of different sizes it is necessary to handle the spreadsheet differently from the common use. We can make the hypothesis that common scheme of the calculation of the average used by some students comes from the basic practice of calculation of average, socially shared and resulting in a socio-cultural obstacle.

Association between the didactical obstacle present in French curricula, the socio cultural-obstacle about the calculation of average, and the constraints of orders of the artefact constitutes an obstacle of a new kind that we propose to call 'instrumental obstacle.'

CONCLUSION

The use of several theoretical frameworks (Lagrange *et al.*, 2003) to describe and analyse the phenomenon of learning in instrumented situations has led us to the construction of the notion of instrumental obstacle.

I am currently further exploring (Chiocca, 2004) the reverse phenomenon when mathematical knowledge creates obstacles in the use of spreadsheets in particular with other obstacles and I am studying problems with the use of spreadsheet.

Finally, there is the potential to explore other dimensions. For example:

- Does other kinds of artefacts use scheme have a link with a socio cultural obstacle?
- Do obstacles exist as a result of the association between an ontogenetic obstacle, artefact use scheme and organization constraints?

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