4. STATISTICAL THINKING IN A TECHNOLOGICAL ENVIRONMENT

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"I learned a lot from my teachers, and even more from my colleagues,
but from my students - I learned the most." (Talmud)

BACKGROUND

Traditional Israeli junior high school statistics usually emphasizes computation and neglects the development of a broader integrated view of statistical problem solving. Students are required to memorize isolated facts and procedures. Statistical concepts rarely originate from real problems, the learning environment is rigid, and, in general, there is just one correct answer to each problem. Even when the problems are real, the activities tend to be "unreal" and relatively superficial. The only view of statistics students can get from such a curriculum is of a collection of isolated, meaningless techniques, which is relatively irrelevant, dull, and routine. Many teachers ignore the compulsory statistics unit. The teachers maintain that there is no time, or that there is pressure to include "more important" mathematic topics, as well as lack of interest and knowledge. We have developed a statistics curriculum (Ben-Zvi & Friedlander, 1997) in an attempt to respond to the need for more meaningful learning of statistics and have incorporated the use of available technology to assist in this endeavor.

THE POTENTIAL OF TECHNOLOGY

Technology provides the opportunity to create an entirely new learning environment, in which computers can be used as tools in problem solving and to foster conceptual development. Thus, students can take a more active role in their own learning, by asking their own questions and exploring various alaternatives to solving them (Heid, 1995; Lajoie, 1993). The use of computers allows students to pursue investigations, can make students less reliant on their teachers, fosters cooperation with fellow students, and can provide students with feedback on progress. We would suggest that the openness of a computerized environment can push students to become intelligent consumers, because they are "forced" to choose from the available tools and options. Thus, computers have made the creation of graphs and tables acceptable problem solving tools, in addition to the more traditional numerical and algebraic methods.

Computers can provide a wider experience with data manipulation and representation, compared to traditional class work (Biehler, 1993). Real situations produce large data bases, which are hard to handle without a computer and which offer many opportunities for investigation using a variety of methods. Also, students have to learn to perform in conditions of temporary or extended uncertainty, becuase they often cannot predict the tool's limitations. When students begin an investigation by posing a question and collecting data, they are unlikely to predict any obstacles ahead of them, such as the wrong data type, missing variables, or tabulating difficulties (Hancock, Kaput, & Goldsmith, 1992); thus, one of the things they have to learn is to evaluate progress and persevere.

Whereas students in the traditional statistics curriculum are able to "plant a tree," students in the computerized curriculum are able to "plant a forest and plan for reforestration." Using software that allow students to visualize and interact with data appears to improve students' learning of data analysis concepts (Rubin, Roseberry, & Bruce, 1988).

Thus, the creation of a technological learning environment should have considerable impact on the contents of the statistics curriculum, and should be accompanied by a broadening of its focus toward an emphasis on conceptual understanding, multiple representations and their linkages, mathematical modeling, problem solving, and increased attention to real-world applications (National Council of Teachers of Mathematics, 1989). In the following sections we introduce the statistics curriculum developed by Ben-Zvi and Friedlander (1997) and preliminary results of our study.

THE STATISTICS PROJECT

The statistics project is a curriculum development and research program, which began in 1993. A statistics curriculum for junior high school (grades 7-9) was developed and implemented using an interactive computerized environment. The project has three components: (1) the development of sets of activities in statistics, (2) implementation in classes and in teacher courses, and (3) research on the learning processes and on the role of teacher and student within this dynamic environment.

In the first year, we developed and tested materials in a few experimental classes and began in-service teacher courses. In the following two years, we extended the experiment to more classes and improved the learning materials based on feedback received. All the work has been accompanied by cognitive research.

The instructional activities promote the meaningful learning of statistics through investigation of open-ended situations using spreadsheets. Students are encouraged to develop intuitive statistical thinking by engaging in activities in which they collect and interpret their own data. A similar approach was used by two projects in the United Kingdom--*Statistical Investigations in the Secondary School* (Graham, 1987) and *Data Handling* for the Primary School (Green & Graham, 1994). In all three projects, the core concept of the curriculum is based on the process of statistical investigation, introduced as Graham's (1987) Pose, Collect, Analyze, and Interpret (PCAI) cycle (see Figure 1). [Pose the question and produce an hypothesis (Stage A), collect the data (Stage B), analyze the results (Stage C), and interpret the results (Stage D)].

Data handling is introduced as "mathematical detective work," in which the student is expected to:

- Become familiar with the problem, identify research questions, and hypothesize possible outcomes.
- Collect, organize, describe, and interpret data.
- Construct, read, and interpret data displays.
- Develop a critical attitude towards data.
- Make inferences and arguments based on data handling.
- Use curve fitting to predict from data.
- Understand and apply measures of central tendency, variability, and correlation.

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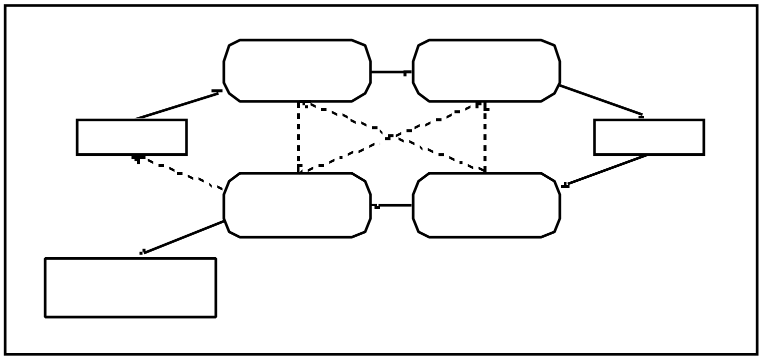

Figure 1: The PCAI cycle for statistical investigation.
(Dotted arrows illustrate possible research paths.)

Our statistics curriculum combines the following two chronologically parallel strands (see Figure 2): concept learning through a sequence of *structured activities* (basic concepts and skills) and a *research project* carried out in small groups (free enterprise).

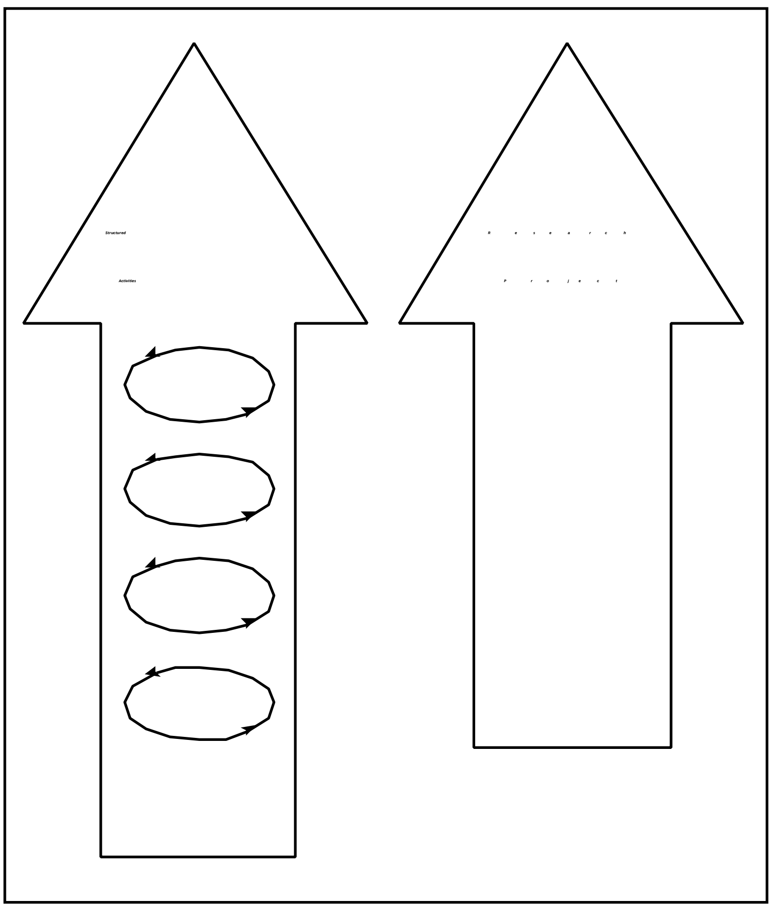

Figure 2: The two strands of the statistics curriculum.

Structured activities

Each structured activity is based on an open-ended problem situation, which is investigated by students in a complete PCAI cycle. The problem situations focus on topics that are of interest to the students (e.g., sports, weather, people's names, salaries, cars) and present new statistical concepts and methods. The students do not always collect the data on their own; sometimes it is given to them in spreadsheet format. Statistical concepts covered include types of data, critical issues in posing questions and collecting data, statistical measures, graphical representations and their manipulation, and intuitive notions of inference and correlation.

In their first meeting, students brainstorm on the word “statistics” and are asked to complete their first small statistical investigation, based on their intuitive notions. This helps to motivate students and gives the teacher some idea of their prior knowledge. During the following investigations, the students are encouraged to make their own decisions about questions to research, tools and methods of inquiry, representations, conclusions, and interpretation of results. Most learning is collaborative; that is, students work in small, heterogeneous groups in a computer-based classroom environment. Students receive assistance from fellow students as well as from the teacher. The teacher is required to create a holistic, constructivist environment (von Glasersfeld, 1984): The classroom becomes an open "statistical microworld" (Papert, 1980, pp. 120-134) in which the student is expected to become a responsible learner. When the class is involved in a structured activity, the teacher introduces the investigation theme, fosters communication among groups, raises questions to encourage thinking, guides the students through technical and conceptual difficulties, and facilitates a discussion of the results. The structured activities are interspersed with more traditional class work, designed to reinforce statistical concepts.

To illustrate the structure of an activity, we will briefly describe one example--the *Work Dispute* in a printing company. The workers are in dispute with the management, who have agreed to a total increase in salary of 10%. How this is to be divided among the employees is a complicated problem, which is the source of the dispute. The students are given the salary list of the company's 100 employees and an instruction booklet as a guide. They are also provided with information about average and minimum salaries in Israel, Internet sites to look for data on salaries, newspaper articles about work disputes and strikes, and a reading list of background material.

In the first part of the activity, students are required to take sides in the debate and to clarify their arguments. Then, using the computer, they describe the distribution of salaries using central tendency measures, guided by the position they have adopted in the dispute. The students learn the effects of grouping data and the different uses of averages in arguing their case. In the third part, the students suggest alterations to the salary structure without exceeding the 10% limit. They produce their proposal to solve the dispute and design representations to support their position. Finally, the class meets for a general debate and votes for the winning proposal. Thus, throughout this extended activity, students are immersed in complex cognitive processes: problem solving with a “purpose” in a realistic conflict, decision making, and communication.

Research project

The research project is an extended activity, which is also performed in small groups. Students identify a problem and the question they wish to investigate, suggest hypotheses, design the study, collect and analyze data, interpret the results, and draw conclusions. At the end, they submit a written report and present their main conclusions and results to fellow students and parents in a "statistical happening." The teacher schedules dates for each stage, guides the students individually to scaffold their knowledge, and actively supports and assesses student progress. Some of the topics students have chosen to investigate include: *superstitions among students*, *attendance at football games*, *student ability and the use of Internet*, *students' birth month*, *formal education of students' parents and grandparents*, and *road accidents in Israel*.

The structured activities supply the basic statistical concepts and skills (which are then applied in the research project) and allow students to get acquainted with the PCAI cycle, the computerized tools, and methods of investigation. The research project motivates students to become responsible for the construction of their knowledge of statistical concepts and methods of inquiry, and provides them with a sense of relevancy, enthusiasm, and ownership.

Thinking processes

During the three years of experimental implementation, we analyzed student behavior using video recording, classroom observations, student and teacher interviews, and the assessment of research projects. The main objective of this paper is to describe some of the characteristic thinking processes observed. Although our data describes all phases of the PCAI cycle, we will concentrate on the last two stages--data analysis and interpretation of results. We present the patterns of statistical thinking in four modes, which were evidenced in all the experimental classes. We are still investigating whether or not these developmental stages are hierarchical and whether or not students go through these stages linearly.

Mode 0: Uncritical thinking

Spreadsheets are powerful user-friendly tools, which allow students to generate a wide variety of numbers, statistical measures, and, more importantly, colorful and "impressive" graphs in large numbers, quickly and easily. As a result, at the initial stage, students are excited by the technological power and exercise it uncritically. Many students explore the software's capabilities, ignoring instructions or any particular order of steps. Their choice of data presentation is based on its extrinsic features, such as shape, color, or symmetry, rather than its statistical meaning. Thus, in this mode, graphs are valued as aesthetic illustrations, rather than providing means for analyzing data.

Students ignore the patterns suggested by their graphical representations, relate to some obvious or extreme features only, or fail to analyze their graphs altogether. Statistical methods are perceived as meaningless routines that must be performed to please the teacher, rather than useful tools that allow one to analyze and interpret data. The default options of the software are frequently accepted and used uncritically, leading to wrong or meaningless information. For example, in order to compare the population of different countries, Na. and Ne. created a bar chart. Since China was included, the default option chose a scale on the y-axis which led to a useless chart. The ease of producing graphical representations led some students to prefer quantity over quality. For example, I. and R. presented 18 bar charts in a project about models and prices of cars in their neighborhood. Each chart presented information about six models. Thus, they were not able to make an overall analysis of any of their research questions.

Mode 1: Meaningful use of a representation

Choosing a representation from a variety of available options is a critical process in statistical analysis. An appropriate representation may reveal valuable patterns and trends in the data, supply answers to questions, and help justify claims.

Two of the typical features of Mode 1 include the following:

Students use an appropriate graphical representation or measure and can explain their choice. The reasons for favoring a representation type are based on intrinsic features, such as the type of data, the method of its collection, or the research questions.

Students are able to perform modifications and transformations of the representation in order to answer and justify their research questions and interpret their results. They reflect on their statistical analysis, identify and perform changes of scale, order of variables and titles, according to their needs.

In Mode 1, students use statistical techniques with a sense of control, reason, and direction. They are able to make changes in their graphs, but tend not to go back and reorganize their raw data in order to achieve a better representation and to draw further conclusions. In other words, students perform well *within* the stage of data analysis (Stage A in the PCAI cycle), but do not make the necessary connections with the C or I stages. We also found that students who operated in Mode 1 ignored numerical methods of data analysis, and mainly justified their inferences graphically. Although students may use many representations (e.g., different types of graphs), these are merely a transformation in shape, rather than adding any additional information or new interpretations. Typically, the student uses representations meaningfully with an interpretation of results, but there is minimal and poor connection between the PCAI stages.

Mode 2: Meaningful handling of multiple representations: developing metacognitive abilities

In Mode 2, students are involved in an ongoing search for meaning and interpretation to achieve sensible results. They make decisions in selecting graphs, consider their contribution to the research questions, and make corresponding changes in the data analysis with a variety of numerical and graphical methods. The process of making inferences and reflecting on the results obtained may lead to the formulation of new research questions. Students are able to organize and reorganize data (e.g., changing the number of categories, designing frequency tables, grouping data, and analyzing subgroups of data) based on results already obtained. Because students are relieved of most of the computations and graph-drawing load, their learning expands to include task management and monitoring. While working on their investigation, students reflect on the entire process, make decisions on representations and methods, and judge their contribution to the expected results (Hershkowitz & Schwarz, 1996). Two examples of this include the following.

G. and A. looked for patterns in the birth month of the students in the school. They hypothesized that spring is the "favorite" birth season. Initially, they produced a pie chart for each of the eight classes in the school. They tried to detect general patterns by adding up the relative frequencies for the spring months. This did not satisfy them and they decided that in order to "prove" their hypothesis, they must aggregate the data for the whole school. They first plotted a bar chart that summarized the data by months for the whole school. Because this graph did not yet yield the expected result, they reorganized their data into the four seasons and drew a second bar chart. This finally satisfied their search for a proof that the "favorite" season is spring.

A. and L. collected data on the number of years of formal education of the parents and grandparents of the students. They expected to find a "relationship" between the level of education of the two generations. They calculated the corresponding statistics (mean, mode, and median) and presented it properly in a tabular format. They noticed that the numerical analysis showed some patterns. However, they preferred to continue their investigation using graphs. They first plotted bar charts, but the picture was too detailed. Nevertheless, they gained the impression that parents studied more than the grandparents. Because the relationship between the two generations was still not clear, they plotted a three-dimensional graph (see Figure 3), showing each generation on a separate plane. They deduced that "*in this graph one can clearly see that the number of years of formal education for the parents is higher than that of the grandparents*." They also observed that the two distributions show the same pattern; that is, that the two graphs have "*the same 'rises' and 'falls.' It seems as if parents and their children are 'glued' together--each of them relative to their own generation.*" They then used a more conventional method that had been taught in class. They produced a scatter plot, with a least squares line, which showed a very weak correlation between the two variables. This presented them with a conflict, because visually they "saw" what seemed to be a high correlation. They preferred the visual argument and so changed (distorted) their concept of correlation, claiming that "*high correlation means equality between the variables and low correlation means inequality*." They concluded that "*this graph* [the scatter plot] *reinforces our hypothesis that parents encourage their children to study more*...."

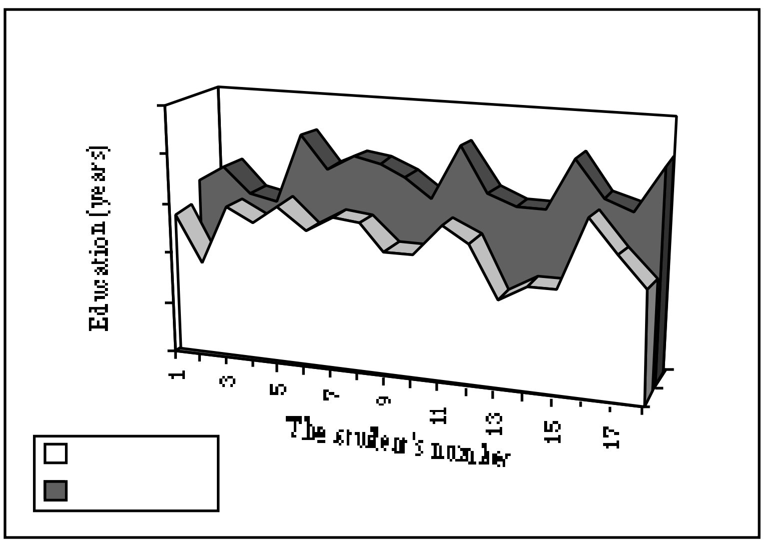

Figure 3: Number of years of formal education for parents and grandparents

In this last case, the students' statistical reasoning was incorrect. However, in both cases we observed that the students reasoned and operated in a flexible and reflective manner. The teams considered ways to verify and prove their initial hypotheses and to convince others of their results. They manipulated their data and favored one representation to another, according to their goal. They were using multiple representations in order to add extra meaning to their research, and were moving independently back and forth in different directions in the PCAI cycle.

Mode 3: Creative thinking

Sometimes, in their search for ways to present and justify ideas, students decide that an uncommon method would best express their thoughts, and they manage with or without computers to produce an innovative graphical representation or method of analysis. Naturally, this type of behavior occurs less frequently. However, an example of this is found in the case of E. E., in grade seven, was investigating the frequency of road accidents in Israel over a period of several weeks, using official figures (Israel Central Bureau of Statistics, 1993). After posing a variety of research questions, he found an interesting relationship between the proportion of accidents involving different types of vehicles and their proportion of the Israeli vehicle population. He plotted a scatter graph of these proportions (see Figure 4). E. added a diagonal line from the origin to the opposite corner, looked at the result, and claimed:

"If a type of vehicle is below the diagonal, it can be considered safer then a type that is above...One can see that the public bus company's campaign to use buses, because of their safety, is not necessarily true, since their share in the total number of accidents (about 4%) is almost twice their share of the country's vehicle population (about 2%)." However, E. concluded his argument by noting that "I need to check the mean annual distance travelled by each type of vehicle, to be sure that my conclusion is true."

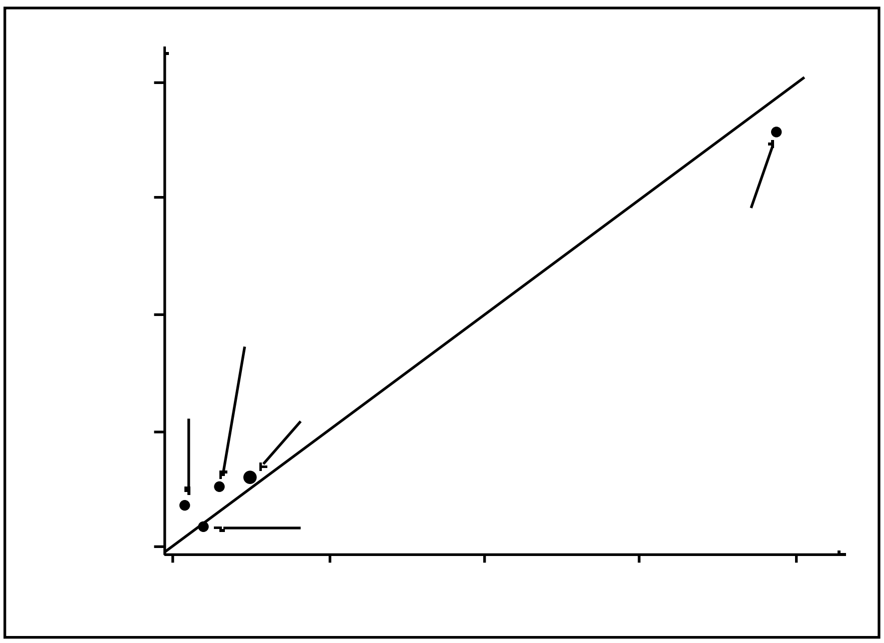

Figure 4: Car accidents in Israel -- E.'s project

E.'s project shows evidence of two novel cognitive modes. The first is E.'s idea to partition the plane and his understanding of the meaning of what he had done. Second, E. identified the need for more data to support a sound conclusion. The first mode reveals flexible unconventional interaction with the graphical representation, and the second reflects his statistical concept understanding.

DISCUSSION

Several aspects are considered that relate to the interaction between the learning environment and the students' styles of work and modes of thinking described above.

Cognitive load

The use of computers shifts the students' cognitive load from drawing graphs and calculations to activities that require higher-level cognitive skills. Thus, students can apply their cognitive resources to operating the software, applying statistical ideas and methods, and monitoring statistical processes. As the cognitive load increased, some students preferred to choose a familiar method rather than looking for alternative methods of analysis and interpretation. In some cases, the computer's graphical capabilities, and the ease of obtaining a wide variety of representations, diverted student attention from the goals of the investigation to some extrinsic features of the tool.

Experience in statistical investigations

The students' lack of experience in conducting a statistical investigation causes, among other things, difficulties in gathering data, tabulating it in spreadsheet format (Kaput & Hancock, 1991), and inefficiencies in analysis methods. Students often failed to foresee the consequences of their strategies on the following stages of statistical problem solving. As they gained experience, they overcame some of these difficulties and were able to connect their goals and ideas to a suitable method of investigation.

Context of investigation

The context in which the original research question is embedded also affects the nature and statistical methods of student work. For example, as reported in the TERC project (Hancock et al., 1992), deep affective involvement and preconceptions related to the context of the planned investigation may lead some students to ignore statistical ideas and draw irrelevant conclusions. Similarly, in our experience, some topics enable the investigators to "take-off" to a higher mode of thought, whereas others leave their performance at a descriptive level. If, for example, the students' question is in a descriptive format (i.e., How many...? Who is the most...?), it may not encourage them to use higher cognitive modes, whereas a question about the relationship between two variables is likely to do that. The teacher can play a significant role in directing students to a potentially stimulating context, and in expanding and enriching the scope of their proposed work.

Combining structured investigations and individual projects

Work on structured statistical investigations, with a given set of data and research questions, helps students to gain experience in data analysis, in the application of statistical concepts, and in the process of drawing inferences. Work in parallel on individual projects allows students to experiment, restructure, and apply, in a creative and open manner, the ideas and concepts learned. This combination of the two strands stimulates students to progress in their use of statistical methods, modes of thinking, and reflection. They become aware of the wide variety of investigation strategies and possible interpretations of results, and finally, they learn to communicate their ideas in written and oral reports.

Teacher actions

An immediate consequence of working in a technological learning environment is that the teacher has more time to meet students on an individual basis, thereby understanding their needs better. On the other hand, the teacher loses some control; that is, the teacher is unable to monitor every detail of the students' actions. Teachers cease to be the dispensers of a daily dose of prescribed curriculum and must respond to a wide range of unpredictable events. In the initial stages, the teacher has an important role: Students need to be encouraged to use critical thinking strategies, to use graphs to search for patterns and convey ideas, and to become aware of the critical role of choosing an appropriate representation and analysis design. Students also need teacher guidance toward a potentially rich context and reflective feedback on their performance. As students become accustomed to the new setting, gain experience in posing more sophisticated research questions, and refine their methods of work and thought, the teacher's role changes from active instructor to fellow investigator.

QUESTIONS FOR FURTHER RESEARCH

We believe that the learning environment described above and the proposed framework for thinking modes in learning statistics may be useful to statistics educators, cognitive researchers, and curriculum developers.

We would like to suggest several further questions that we have not yet considered.

- Does the student who learns statistics in a technological environment undergo a developmental process or are the methods of work described above a set of discrete and not necessarily hierarchical modes?
- What are the contexts of investigation that foster higher-level statistical thinking?
- How do students of various learning abilities respond to the proposed learning environment?
- What are the teacher actions that stimulate students to use meaningful multiple representations and to develop metacognitive abilities and creative thinking?

It would be interesting to hear from others whether their observations with similar materials replicate the suggested categorization of student thinking modes. In so doing, perhaps some of the questions regarding development may be answered. We to investigate the data gathered over the last three years to answer, if only partially, some of these questions.

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