CHANGING THE UNDERSTANDING OF PROBABILITY IN TALENTED CHILDREN

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This paper refers to the effectiveness of an instructional sequence of lessons related to Probability, which were implemented to 11 to 13 years old talented children. This instructional sequence as part of a one semester math course, aims to develop children's understanding of probability. Contextualized situations involve decision making into uncertainty, confrontation of intuition with experimental probability, and adhesion to theoretical probability. The pretest included equiprobable and non-equiprobable, independence and dependence concepts. Lessons were focused on students´ work, especially in pairs and small groups. Experimental situations always included plenary discussion, where the teacher acted as moderator. The posttest gave evidence of overcoming preconceptions and adhesion to the principles of theoretical probability.

BACKGROUND

In Chile, education as an improvement of human capital is priority for reasons of competitiveness and fairness, and is why the development of excellent human resources is fundamental in the country (Cox, 2003). The results of the national system of measurement of educational quality tests give clear evidence that the children from the poorest social level obtain lower learning outcomes. On the other hand, independent of social level, academic talent manifests itself in an eight to ten percent of the population, and research says that talents should be developed early, so are not lost. In this context the Universidad Católica de Valparaíso developed a program BETA PUCV to support the development of academic talented children from vulnerable socio-economic sectors.

Moreover, over the past few decades the national school curriculum has incorporated the topics of statistics and probability with increasing emphasis. In the 1980s these topics were considered only in the last year of secondary education, however, towards 1990, were included in secondary education, and settling the groundwork during the second cycle of basic education. Now, however, the new curriculum implemented this year, 2009, as the result of international trends, introducing the teaching of statistics and probability from grade 1 of elementary education through grade 12 of secondary education, all schooling.

THE PROBLEM

The inclusion of probability in the curriculum presents an enormous challenge to the educational system, as in other countries. On one hand, the teachers in-service are faced with the task of teaching this content they have not necessarily studied in their initial training, and secondly, the teacher training system assumes the mission to provide these skills to teachers without necessarily having specialists with appropriate academic expertise. In fact, the area of didactics of statistics is emerging and recognized the lack of knowledge grounded in experience as teachers to transfer to the system.

This work aims to give an account of the process and the results of the design and implementation of a sequence for learning probability, based in the presentation of a problem situation, intra-group works, and emphasis on intergroup and plenary discussion, so children reach final conclusions from their arguments and/or discussion among their peers, (Isoda & Olfos, 2009). The main premise is that children build mathematical knowledge, probability, through solving problem situations within the framework of their strategies and argumentations, both individually and collectively.

While this is a six week experience with talented children, the program could be implemented at a slower pace for children with normal abilities, this allows for the child to face challenging questions and develop more independent thinking and communication skills, discussing their findings and conclusions, together with forms of statistical thinking.

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EXPERIENCE AND SUBJECTS

The research was conducted with a group of 22 children from level six and seven, whose ages ranged from 11 to 13 years, of the Valparaíso region, within the framework of a course in the PUCV BETA program of the Universidad Católica de Valparaíso, providing educational opportunities of high quality for the integral development of children from academic talents of vulnerable socio-economic sectors. The group was formed by 8 girls and 14 boys, who had not received formal education on the topic of probability, and according to their age, were able to develop their own logic operations of combinatorial thinking, possessing the necessary knowledge of fractions and ratio comparisons, required to provide answers to the problems concerning probability.

The Probability class was conducted during 16 hours taken over six weeks. The objective was to deliver notions of Combinatory and Probability; by performing random experiments build a language of chance, to discover through experience experimental probability, and compared with theoretical probability. The classes raised classical problems and situations and others such as those proposed in the working guidelines for the introduction of probability and combinatory available on the website of the NCTM (2000).

The classes informally introduced various procedures and concepts. The deterministic approach was developed from the combinatorial analysis, use of tree diagrams, and Pascal's triangle. Then, they were introduced to non-deterministic events through the implementation of randomized experiments, introducing the concepts of event, sample space, elemental and compound event, impossible event, equally likely events, probability as a measure of random, relative frequency, Laplace’s rule, and law of large numbers. Experimental situations first predicted or estimated to test their own intuition; then performed the analysis and visualization of the results, with discussion and using tree diagrams or frequency graphs, aimed to generalize and finally confront the first intuition.

METHODOLOGY

At the beginning of the course a test was applied to assess students’ probabilistic reasoning and compare it to the level achieved after their learning experience during the Probability course. The time interval between the two applications of the questionnaire was six weeks. The questionnaire takes as reference three items from the study by Maury (1988) “Procedures dans probabilistes problems resolution” , three items from the thesis for the Master of Didactics of the Sciences (Cuadra, 1997), two items from Carmen Díaz questionnaire (2007), and one item modified by the author from the literature. The questionnaire consists of nine items, seven simple probability (independent events) and two items of dependent events. The content assessed by each item is displayed in Table .

Table 1. Content assessed

Content	Items
Favorable and unfavorable cases, simple probability	1
Using fractions or ratios according to case to be favorable or not, likely simple	2
Comparison of favorable and unfavorable cases, simple probability	3 & 7
Gambler's fallacy with equiprobable events, simple probability	4 & 6
Determination of conditions equiprobability	5
Calculate a conditional probability in a synchronic situation	9
Calculate a conditional probability in a situation diachronic	8
Calculate a joint probability in dependent event	8 & 9

RESULTS

After applying the pretest and the posttest to 20 of the 22 participating children, analysis was conducted to determine students’ progress in probabilistic reasoning. Correct answers were awarded one point, and zero points if partly right or wrong. Scores by 20 subjects who participated both in the pretest and the posttest are displayed in Table 2.

Table 2. Subject scores

Item	1	2	3	4	5	6	7	8	9
Correct answers Pretest	14	9	12	2	15	2	2	0	0
Correct answers Posttest	18	14	17	15	18	14	10	0	4

This summary of responses highlights the high number of erroneous or partially correct responses for items four and six. Items four and six correspond to contexts about fair coins and dice:

> Item 4: "It flips a fair coin". In the first five flips has been obtained head. In the sixth flip, what will it be, head or tail? Justify.
> Item 6: A fair die is rolled. The first roll is "1". The next seven rolls are all "5". In the ninth roll, what number will it be? Justify.

These two items intended to determine whether students are influenced by previous results, both have in common the work and independence of events. The responses show the fallacy of the player, so that repeated independent trials, mistakenly believed that past events affect future ones. In the wrong answers of children shows that they are influenced by the data given in the description of the experience, or in the words of Fischbein, these students do not have developed intuitions about the independence of elementary events. In the pretest answers both, Piaget (1975) and Fischbein (1975) reported for preschool students. Among the arguments given are: the spatial location of objects ("you should not change the amount of balls, but the position. In that position is likely to draw blue, then red"), the belief that random events can be controlled by the person performing the experiment ("depends on which side put the coin"), and the perception of object representation ("tail because in the drawing stamp out ").

Among the incorrect answers presented in the pre-test is "head, as it is the tendency, certainly the coin has some", and the same student, 13 years old, following the instruction says "is the same probability of both, head as a tail, experimental results are different from the theory, 1/2 = 1/2". Another student, 11 years, erroneously argued "head, because if left five pitches and head may be very possible to have head again" then the instruction, the argument is "This case is equiprobable because although head has left the 5 previous releases also the sixth launch may head or tail out because they have the 50%".

Items eight and nine relating to dependent events present zero correct answers. In item eight, although the choice of children was correct, their justifications were not relevant. These low results are consistent with the fact that it was not possible to treat dependent events in the amount of time assigned to the course content.

In spite of the items three and seven measured the same, only that used different contexts, roulettes wheels and bags respectively, we conclude like that Maury (1988) and also by Cuadra (1997), the use of roulettes favours an adequate probabilistic reasoning.

The high proportion of children who correctly answered both, the pretest and posttest to one, two, three and five, items shows that these children have a good intuition probabilistic, in the context of his academic talent. The most significant advance made by these children after the Probability classes are the quality of their written arguments in the questionnaires. For example, the justification for a student, 11 years 5 months, in the pretest was "is most likely to go blue because there are more blue than red segments", after 6 weeks the same student argues comparing fractions and percentages, "blue because it has more probability, P (blue) = 3/5 = 60%, P (red) = 2/5 = 40%,"; a student of 11 years and 4 months, justifies precariously on the pretest, based in the drawing "blue, because it is only supported for a bit there", then the same item argues comparing reasons: "blue because the probability is 3/5 and the red is 2/5".

CONCLUSION

Interpretative analysis on answers and arguments delivered by talented children before and after learning experiences, shows that talented children improve their arguments involving probabilistic reasoning, after a short course of 16 hours of Probability classes. This is a dialectic process in the context of interactions which encouraged discussion and taught students to describe, explain and justify the findings and procedures used, articulating formal arguments of probability theory and justifying with numerical arguments.

The type of arguments provided in the responses by talented students, serve as a model of answers expected in the framework of the realization of the classes for standard students. Thus, the story in detail of this experience is useful for the training of teachers in service, because these may not have been a part of their teacher preparation; they need in-service training and a lot of support in both content and pedagogy.

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