Bayesian parameter estimation for inspiral signals observed by LISA

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▶ Inference on signal parameters

When the gravitational wave signal of a binary inspiral event is observed by LISA, the measurement will always be superimposed with noise from various sources. In order to accurately infer the parameters of the original signal, we follow a Bayesian approach. This means in particular that quantification and derivation of information about the signal parameters is done in terms of *probability distributions*. In order to complete the analysis, the necessary integration over the parameters' *posterior distribution* (given the data at hand) is done using Markov chain Monte Carlo (MCMC) methods.

▶ Noise problems

In the context of LISA, the challenge is that the noise is very complex, that is, it is not simple white noise, but it is a mixture of various sources of *instrument noise* and *background noise*; in particular, the galactic background noise will consist of countless superimposed modulated periodic signals. The problem, then, is to find a sensible model formulation that accurately reflects states of knowledge (and ignorance) about parameters and noise, without ignoring nor explicitly modeling each of the background sources individually.

Due to LISA's setup, its individual 'raw' data outputs will also have highly correlated noises. In order to minimise this effect, we base our inference on the derived *Time Delay Interferometry* (TDI) variables [1].

▶ Wanted:

...a model formulation based on minimal assumptions, that is able to account for the numerous unknown background signals and allows for rigorous inference at reasonable computational cost, without shortcuts or approximations.

▶ Approach

Model the noise in a most flexible way, making use of the *Maximum Entropy* principle so that only minimal assumptions (here: finiteness of the spectrum) enter the specification.

The model specified this way will not only reflect the *randomness* in the noise, but also the *ignorance* about deterministic, but unaccounted for signals within the noise [2].

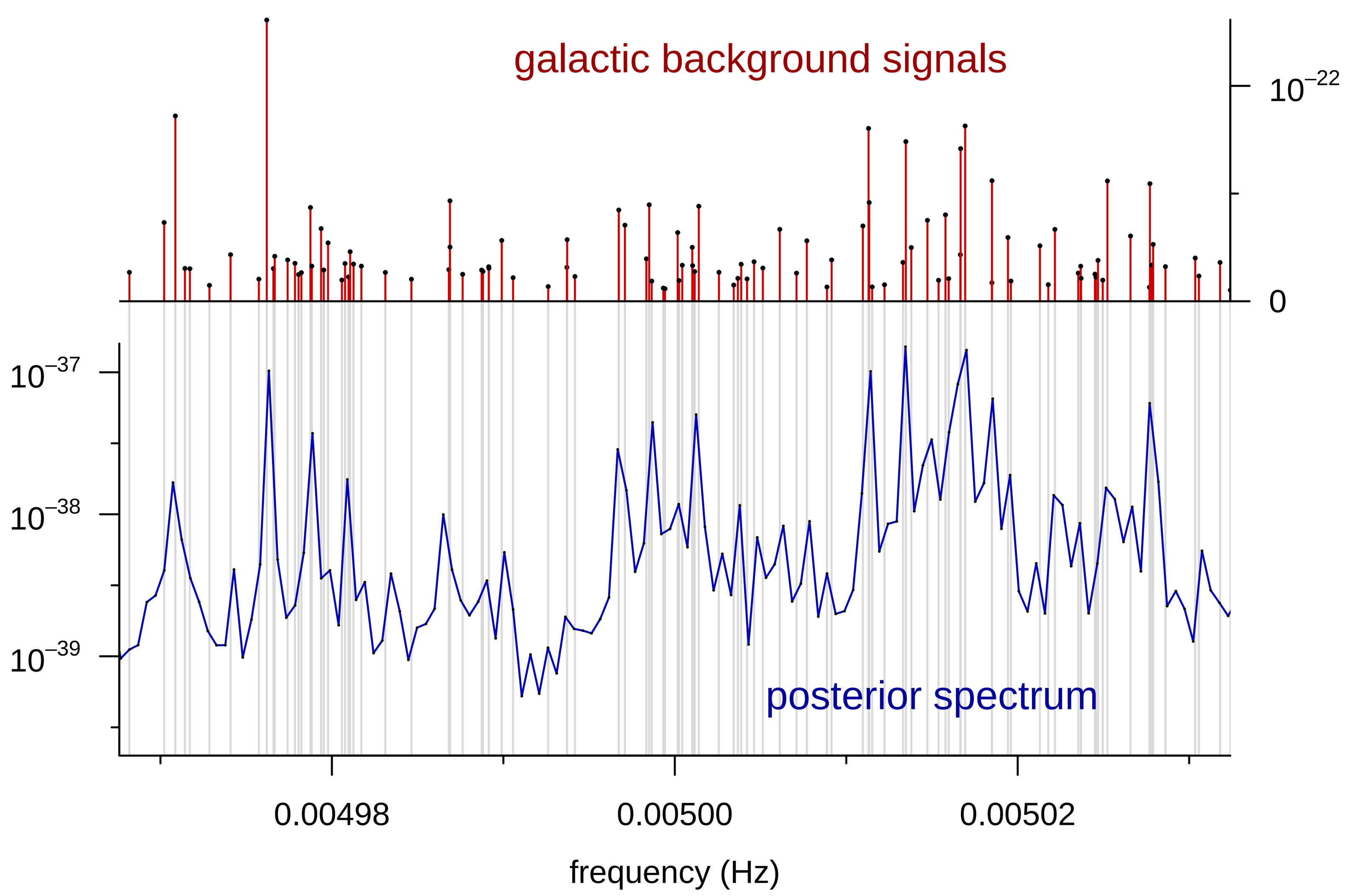
▶ The model

The observed data, measured at discrete timepoints $t_1, \ldots, t_N$, are assumed to be the signal $s_\theta(t)$ (dependent on the parameters θ) plus additive noise $n(t)$. For each t_i, the noise then is defined as:

$$n(t_i) = \sum_{j=0}^{N/2} a_j \cos(2\pi f_j t_i) + b_j \sin(2\pi f_j t_i)$$

where the f_j are the Fourier frequencies, and a_j and b_j follow a Normal distribution with zero mean and *unknown* variance σ_j. This specification adds an extra $N/2$ noise parameters (σ_j) to the model, for which prior information can be supplied, and which are inferred along with the signal's parameters θ. Estimation of each noise parameter σ_j is based on the 'observed' a_j and b_j. Thus, or approach involves:

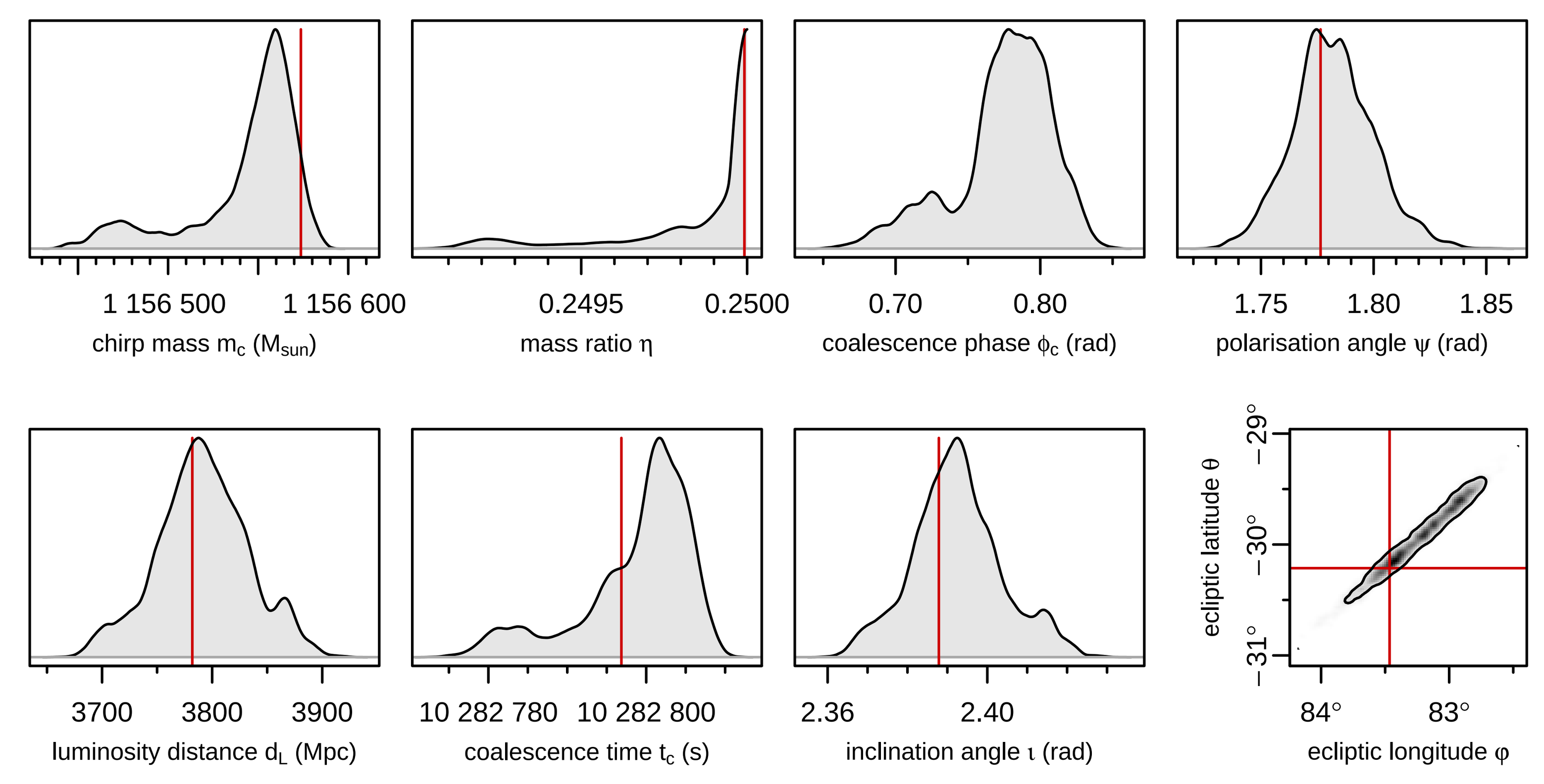
- ➥ a robust and general model, which makes minimal assumptions
- ➥ generalisation of a simple, Gaussian noise model
- ➥ parameters a_j, b_j which correspond to Fourier coefficients
- ➥ σ_j corresponding to noise spectrum
- ➥ a conjugate (conditional) posterior, with easy implementation



(True) frequencies and amplitudes of individual background signals within a narrow frequency band, and how these are reflected in the posterior noise spectrum.

▶ Application to MLDC round 2 data

The model was applied to a simulated supermassive black hole inspiral signal from round 2 of the Mock LISA Data Challenges ('SMBH-1' from challenge 2.2). The signal waveform here is a restricted PN approximation defined by 9 parameters [3]. Monte Carlo integration of the posterior is done using a parallel tempering MCMC sampler in a parallel implementation [4]. For the internal numerical derivation of the detector response to a given signal, the 'LISA Simulator' is used [5].



Posterior densities for the 9 signal parameters. True values are shown in red (except for ϕ_c, due to a different parametrisation).

This general and robust approach will be useful with real data, where the noise properties are not exactly known beforehand but need to be estimated, along with the signal parameters from the same data and across the entire frequency range.

References

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