| Statistics 726 | Assignment 1 | Due: March 21 |
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1. Consider the set of all possible observation vectors

$$\mathbf{x} = (x_1, \ldots, x_n)$$

with inner product defined by

$$\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i \, y_i .$$

Define

$$\mathbf{1} = (1, \ldots, 1)$$

and let $\mathcal{M}$ be the set of vectors of the form $\alpha \mathbf{1}$, and $\mathcal{V}$ be the set of vectors orthogonal to $\mathcal{M}$.

(a) Compute the projection $P_{\mathcal{M}}\mathbf{x}$ of $\mathbf{x}$ onto $\mathcal{M}$ and the projection $P_{\mathcal{V}}\mathbf{x}$ of $\mathbf{x}$ onto $\mathcal{V}$.

(b) In statistical terms, what is the value of

$$||P_{\mathcal{V}}\mathbf{x}||^2 / ||\mathbf{1}||^2.$$

(c) For two vectors $\mathbf{x}$ and $\mathbf{y}$, compute the cosine of the angle between $P_{\mathcal{V}}\mathbf{x}$ and $P_{\mathcal{V}}\mathbf{y}$. What is the statistical interpretation of this value?

2. Plot the autocorrelation functions for the following ARMA models.

(a) AR(2) with $\phi_1 = 1.2$ and $\phi_2 = -0.7$.

(b) AR(2) with $\phi_1 = -1$ and $\phi_2 = -0.6$.

(c) MA(2) with $\theta_1 = 1.2$ and $\theta_2 = -0.7$.

(d) MA(2) with $\theta_1 = -1$ and $\theta_2 = -0.6$.

(e) ARMA(1,1) with $\phi = 0.7$ and $\theta = 0.4$.

(f) ARMA(1,1) with $\phi = 0.7$ and $\theta = -0.4$.

3. Define $Y_t = a\cos(\lambda t + \phi)$ with $\phi \sim \mathrm{U}[0, 2\pi]$. Show that $Y_t$ is (weakly) stationary and compute its autocovariance function. (Hint: Those good-old formulas for the sin and cos of the sum and difference of angles might be useful.) Is $Y_t$ strictly stationary? (You need to justify your answer.)