THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2003
Campus: City

STATISTICS

Time Series Analysis

(Time allowed: TWO hours)

NOTE: Attempt all FOUR questions.
All questions are worth equal marks.
You should allot equal time for answering each question.

1. (a) Define the *autocorrelation function* (ACF) for a stationary time series.

(b) Derive the ACF for the MA(2) series generated by the scheme

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2},$$

where ε_t is a white noise series.

(c) Derive the ACF for the AR(1) series generated by the scheme

$$Y_t = \phi Y_{t-1} + \varepsilon_t,$$

where $|\phi| < 1$ and ε_t is a white noise series.

(d) For the ARMA(1,2) model

$$Y_t = 0.8 Y_{t-1} + \varepsilon_t + 0.7 \varepsilon_{t-1} + 0.6 \varepsilon_{t-2},$$

show that

(i) $\rho(k) = 0.8\rho(k-1)$, for $k \geq 3$ and
(ii) $\rho(2) = 0.8\rho(1) + 0.6\sigma_\varepsilon^2/\gamma(0)$.

(e) Describe, in general terms, the behavior you would expect to see for the *estimated* ACF computed from the set of observations $Y_1, \ldots, Y_T$ generated by the random walk model

$$Y_t = Y_{t-1} + \varepsilon_t.$$

(Hint: Y_t is *almost* an AR(1) series.)

CONTINUED

2. (a) Explain what it means for a times series Y_t to be *stationary* and *mixing*.

(b) Define the *power spectrum* for a stationary, mixing time series and describe what the power spectrum reveals about the time series.

(c) Compute the power spectrum for a white-noise series ε_t with $\mathrm{E}[\varepsilon_t] = 0$ and $\mathrm{var}[\varepsilon_t] = \sigma^2$.

(d) Explain what it means for an operation carried out on a time series to be a *linear time-invariant filter*.

(e) Show how to write the *transfer function* of a linear filter as a function of the filter's coefficients.

(f) The seasonal differencing filter transforms Y_t to $Y_t - Y_{t-s}$ (simple differencing has $s = 1$). Compute the transfer function of this filter.

(g) The seasonal summation filter transforms the series Y_t to $Y_t + Y_{t-1} + \cdots + Y_{t-s+1}$. Show that the seasonal difference filter is equivalent to first applying the seasonal summation operator and then simple differencing.

(h) Using the result of part (g) (or otherwise) compute the transfer function of the seasonal summation operator.

3. The following data set gives the number of accidental deaths occurring monthly in the United States during 1973–1978.

Monthly Accidental Deaths in the U.S.A., 1973–1978

	1973	*1974*	*1975*	*1976*	*1977*	*1978*
Jan.	9007	7750	8162	7717	7792	7836
Feb.	8106	6981	7306	7461	6957	6892
Mar.	8928	8038	8124	7776	7726	7791
Apr.	9137	8422	7870	7925	8106	8129
May	10017	8714	9387	8634	8890	9115
Jun.	10826	9512	9556	8945	9299	9434
Jul.	11317	10120	10093	10078	10625	10484
Aug.	10744	9823	9620	9179	9302	9827
Sep.	9713	8743	8285	8037	8314	9110
Oct.	9938	9129	8433	8488	8850	9070
Nov.	9161	8710	8160	7874	8265	8633
Dec.	8927	8680	8034	8647	8796	9240

(a) In order to investigate possible non-stationarity of the series, the following three plots are produced from the series. What kind of effects do the plots show and how would these effects influence the analysis of the series?

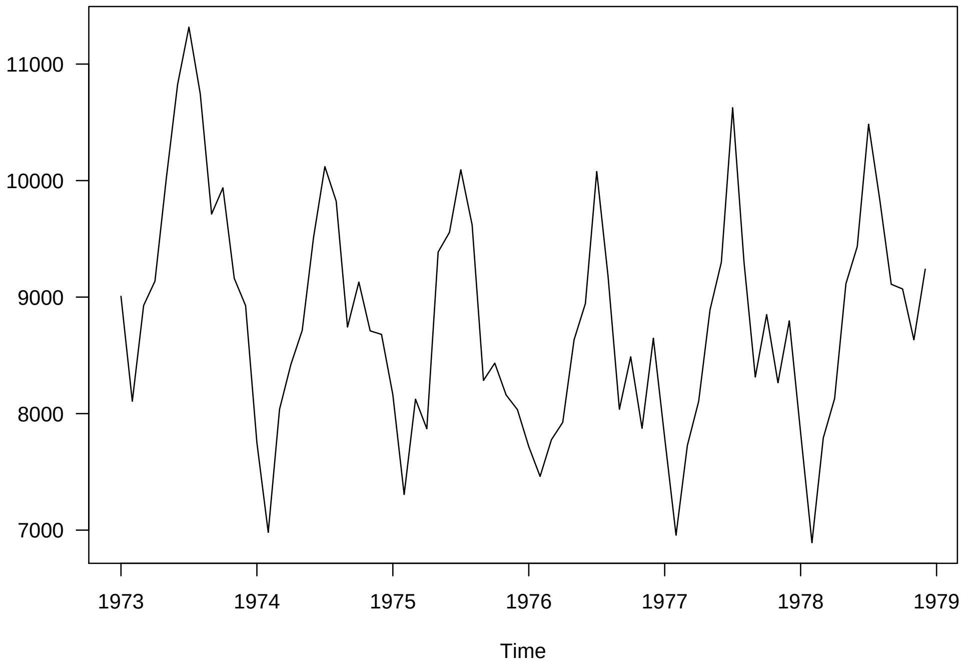


Figure 1: A plot of the original accident time series Y_t.

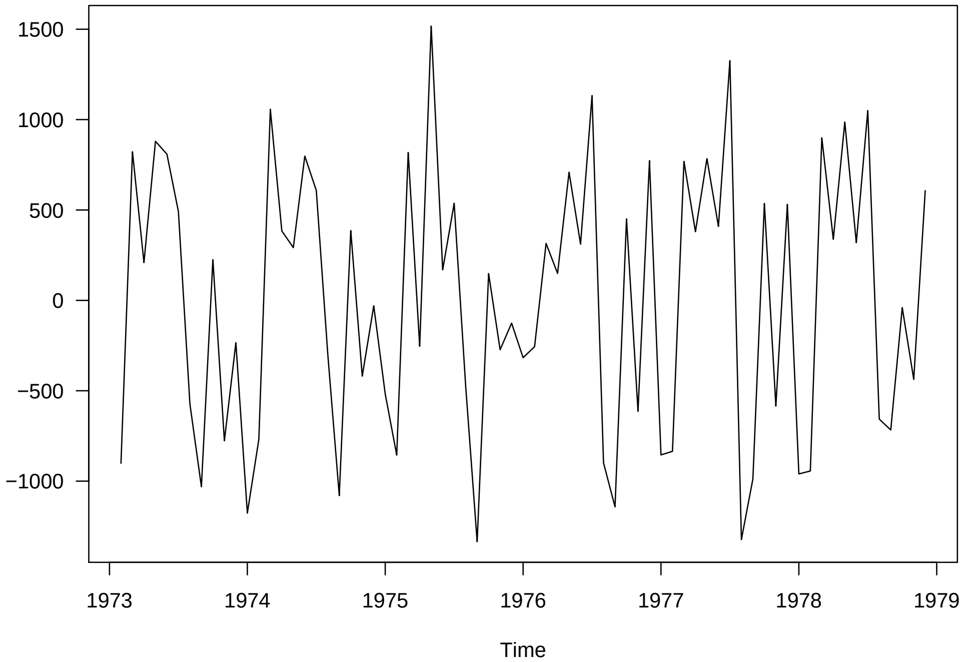


Figure 2: A plot of the differenced accident time series $Y_t - Y_{t-1}$.

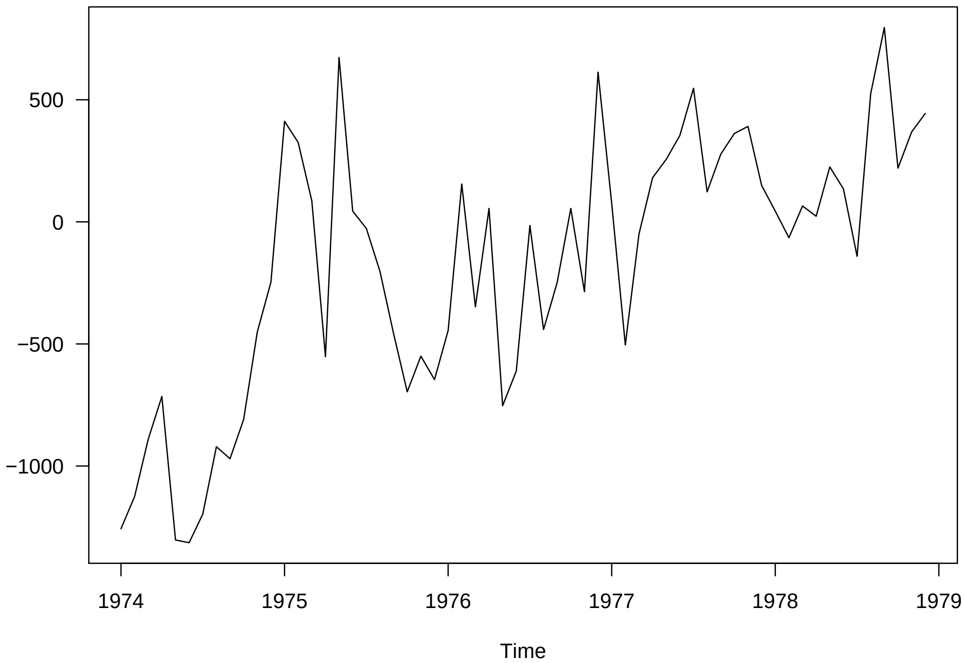


Figure 3: A plot of the seasonally differenced accident time series $Y_t - Y_{t-12}$.

(b) The time series is read into an R data set called `deaths` and the following statements are issued to R.

```
> acf(diff(diff(deaths, 12), 1), 24)
> pacf(diff(diff(deaths, 12), 1), 24)
```

Describe in detail exactly what these statements do.

(c) The following graphs are produced as a result of running the R statements above. Explain what kind of model structure you think that the graphs indicate is appropriate for modelling this series.

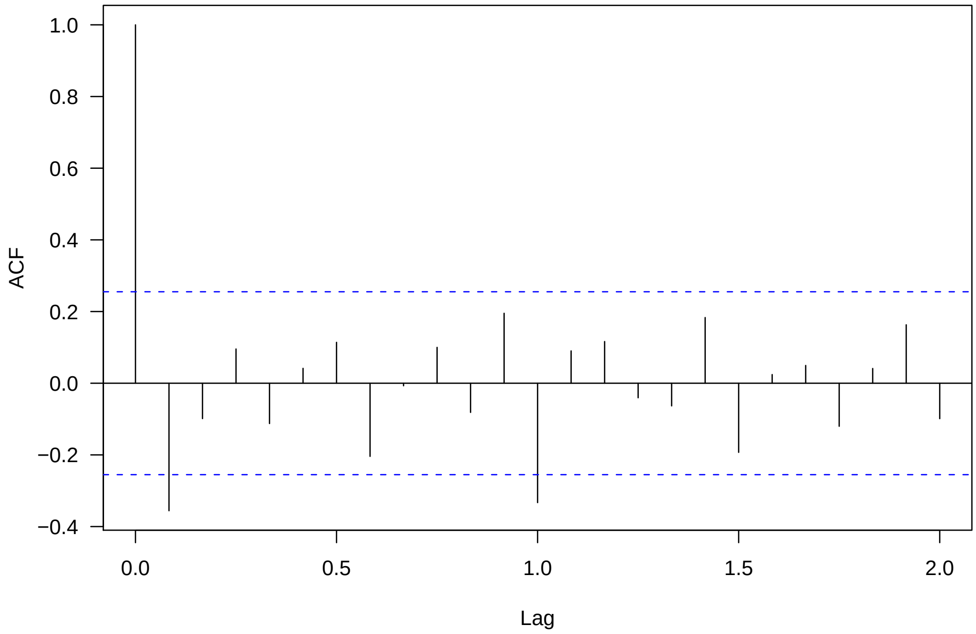


Figure 4: The first plot produced by the R statements.

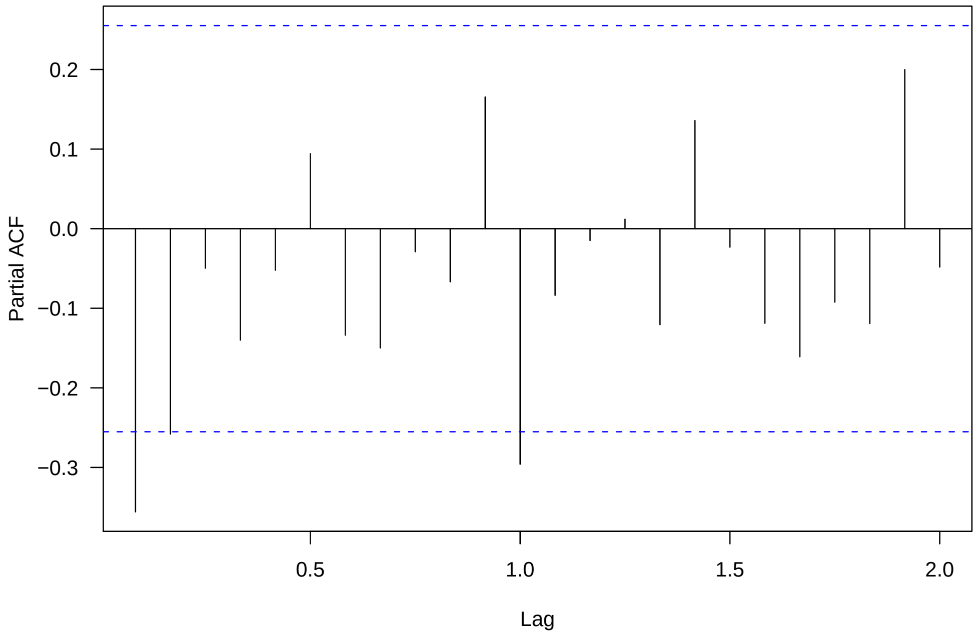


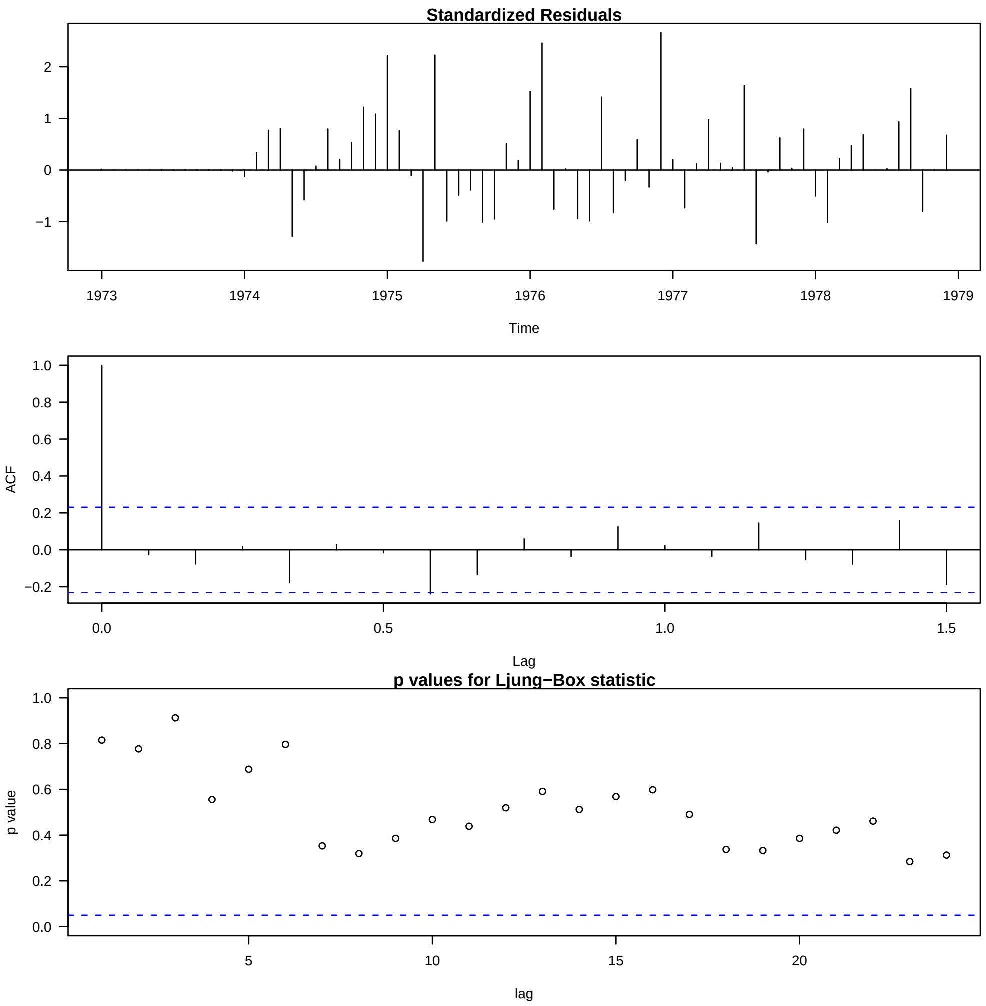
Figure 5: The second plot produced by the R statements.

(d) Using operator notation, write down the **complete** model which should be fitted to the accident data series.

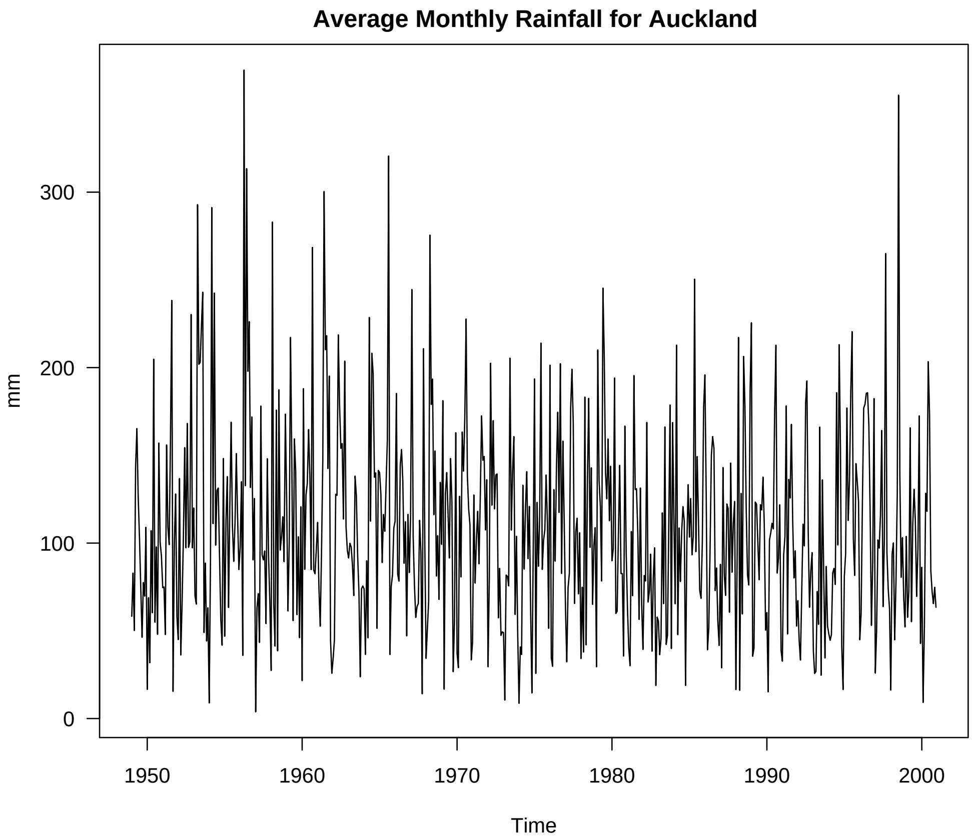
(e) A time series model is fit to the `deaths` series and the following results and plots obtained. Does the model fit well? Give reasons.

```
Coefficients:
       PARAM1   PARAM2
      -0.4264  -0.5584
s.e.   0.1226   0.1787

sigma^2 estimated as 99480
log likelihood = -425.53
aic = 857.06
```



4. The plot below shows Auckland's total monthly rainfall (in mm) for the period from January 1949 up to December 2000.

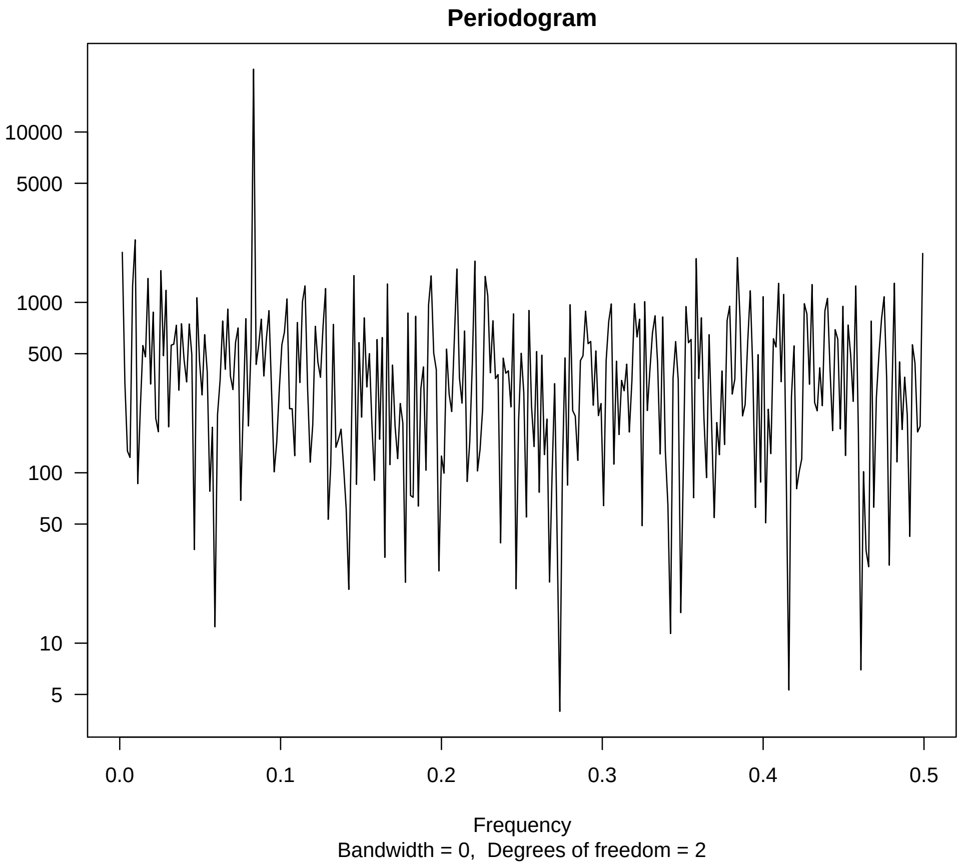


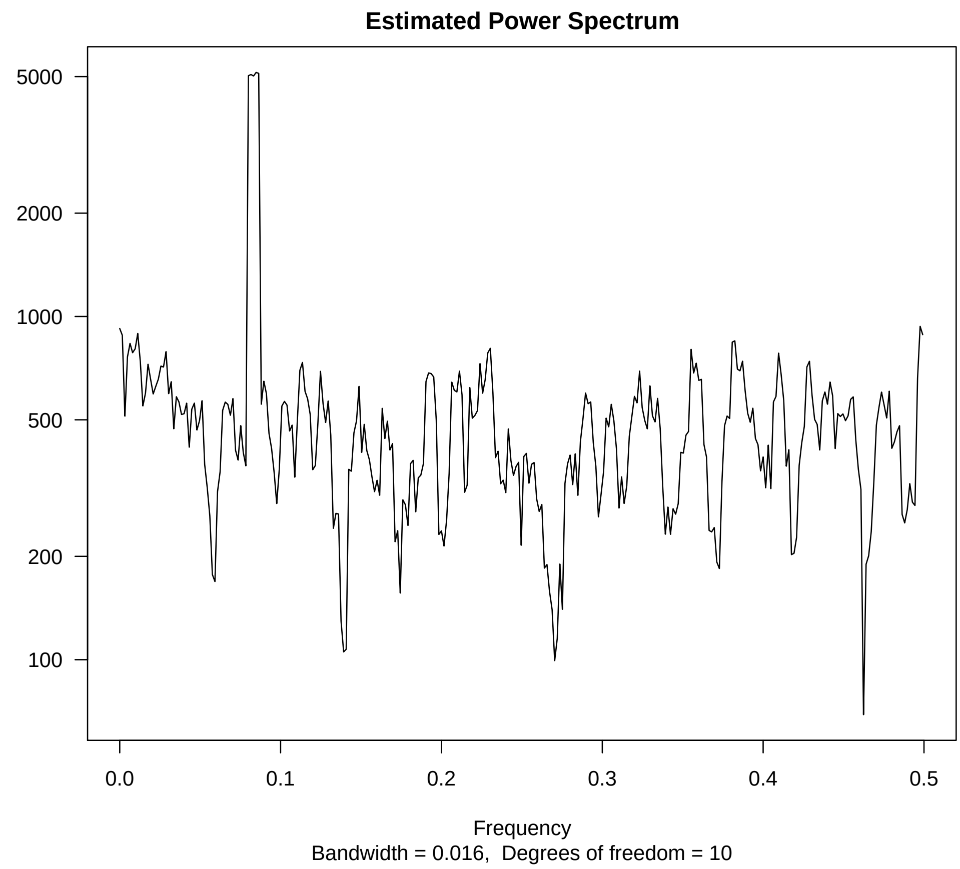
On the following page, there are plots which show the periodogram and a power spectrum estimate for the series.

(a) Carefully explain the terms *bandwidth* and *degrees of freedom* which appear in these plots.

(b) Explain how the power spectrum estimate was obtained from the periodogram.

(c) On the basis of these plots, suggest a suitable model for Auckland's rainfall. (There are one or two subtle details.)


Periodogram
10000
5000
1000
500
100
50
10
5
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
Bandwidth = 0, Degrees of freedom = 2


Estimated Power Spectrum
5000
2000
1000
500
200
100
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
Bandwidth = 0.016, Degrees of freedom = 10

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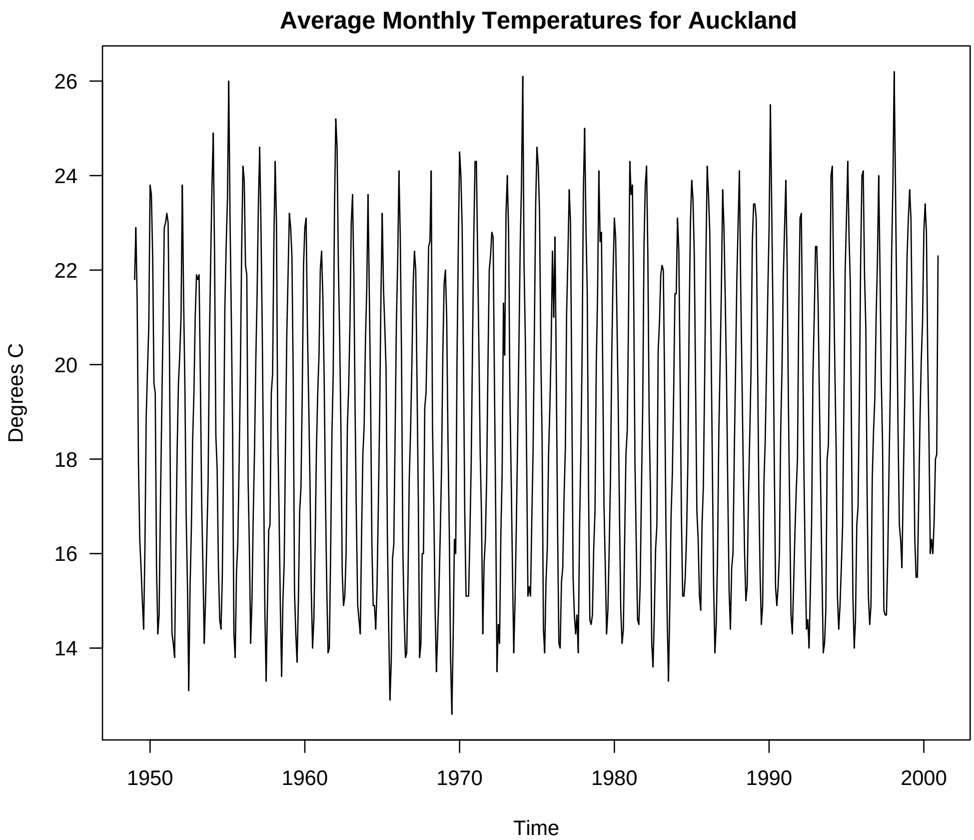
The plot below shows Auckland's average monthly temperature (in $^\circ$C) for the period from January 1949 up to December 2000.

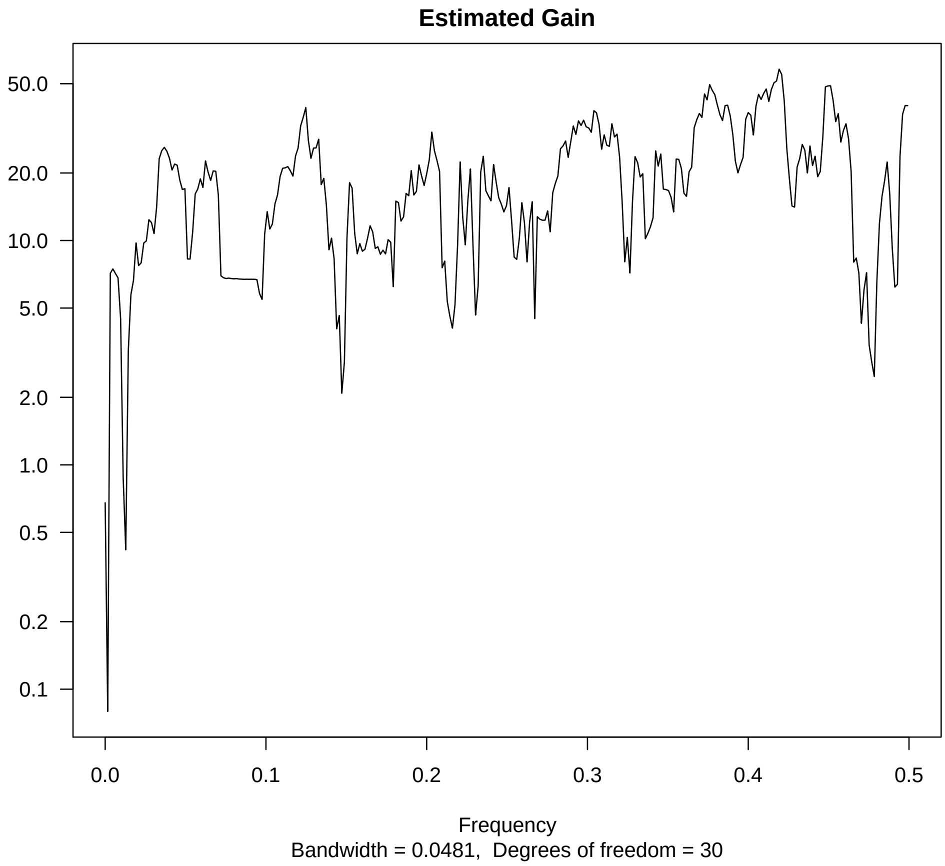


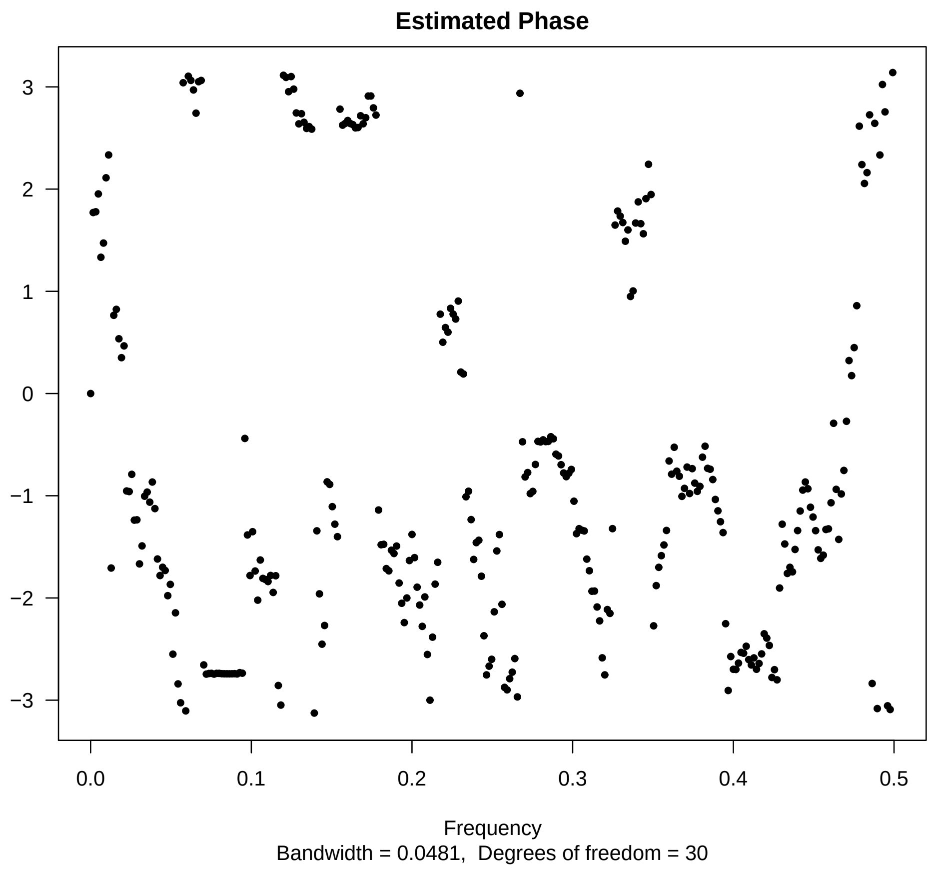
To see how the amount of rainfall in Auckland depends on temperature, a cross-spectral analysis of rainfall on temperature was run. Some of the results of this analysis are shown on the next two pages.

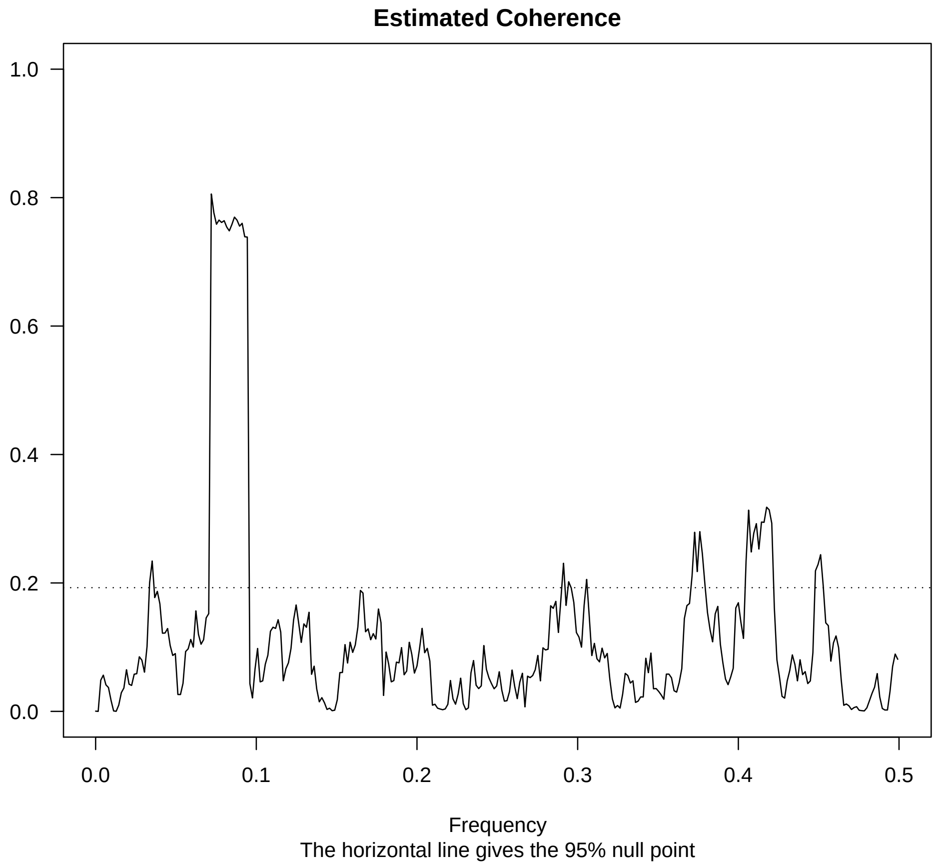
(d) Write down the model used in a cross-spectral analysis.

(e) Comment on the strength of the relationship between temperature and rainfall.

(f) Using what you know about the pattern of temperature and rainfall in Auckland, explain the estimated phase values corresponding to yearly cycles.


Estimated Gain
50.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
Bandwidth = 0.0481, Degrees of freedom = 30


Estimated Phase
3
2
1
0
−1
−2
−3
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
Bandwidth = 0.0481, Degrees of freedom = 30


Estimated Coherence
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
The horizontal line gives the 95% null point