THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2004
Campus: City

STATISTICS

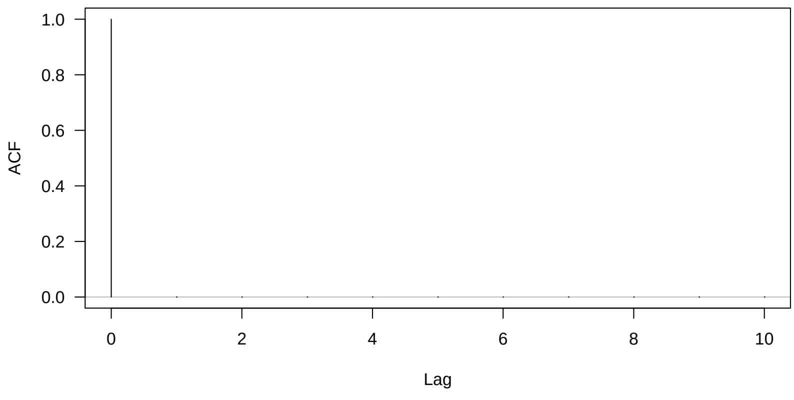
Time Series Analysis

(Time allowed: TWO hours)

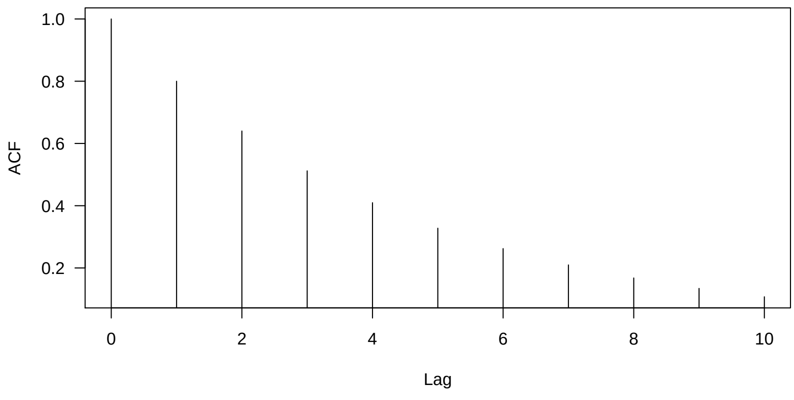
NOTE: Attempt all FOUR questions.
All questions are worth equal marks.
You should allot equal time for answering each question.

1. (a) The following plots show examples of (theoretical) autocorrelation functions for a variety of ARMA models. In each case, give the simplest possible description you can of the underlying model. Explain your reasoning fully.

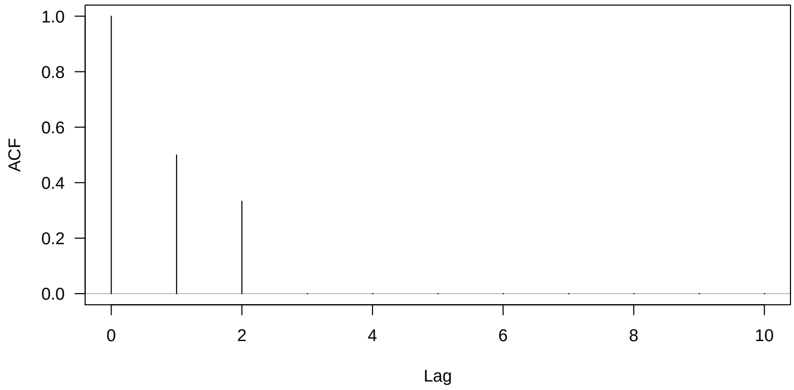
Case (i)



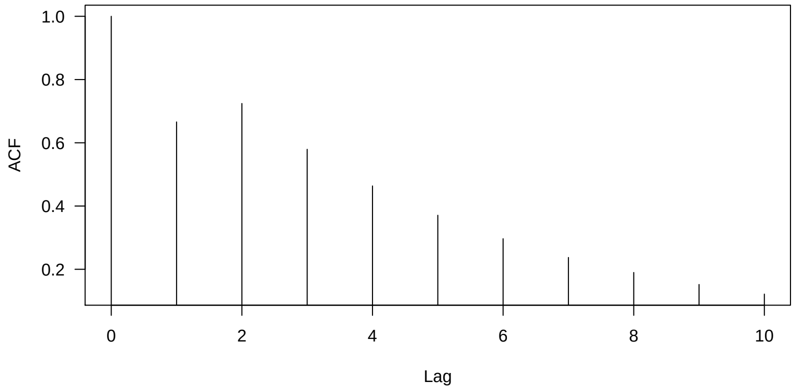
Case (ii)



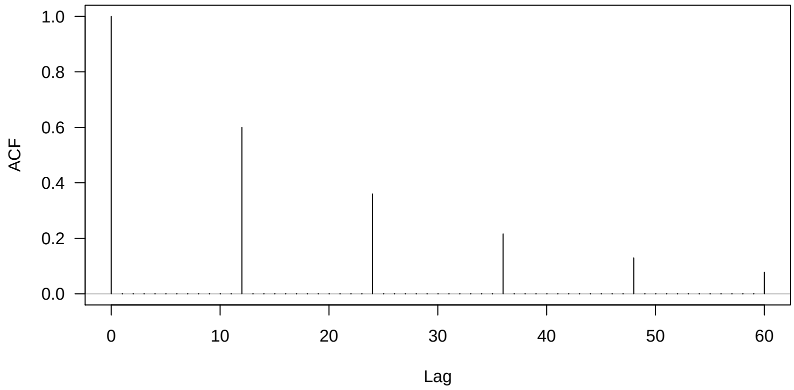
Case (iii)



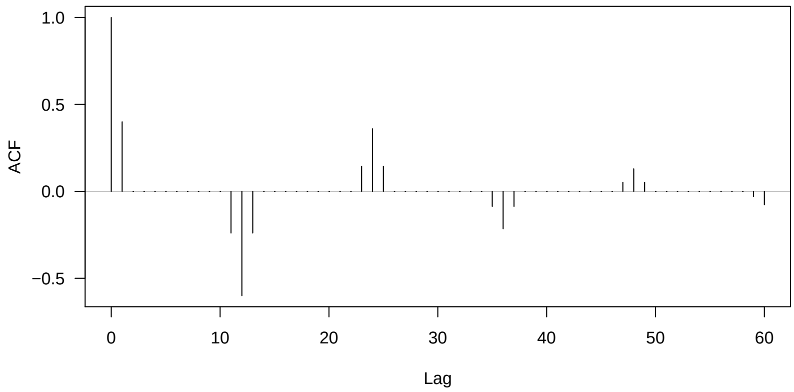
case (iv)



case (v)



case (vi)



(b) Does the equation

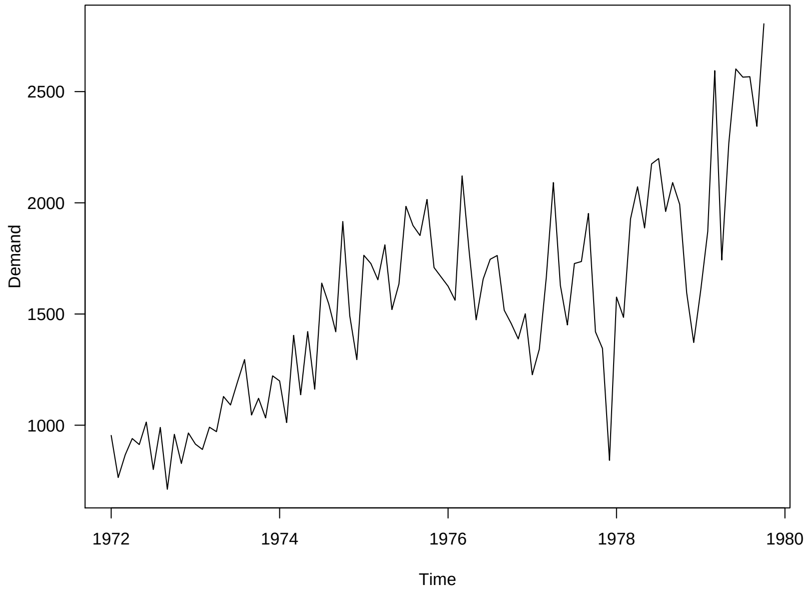
$$Y(t) = \tfrac{1}{2}Y(t-1) + \tfrac{3}{4}Y(t-2) + \varepsilon(t)$$

define a stationary, causal time series? Explain.

(c) Show that for an MA(1) process, the lag 1 autocorrelation $\rho(1)$ satisfies $-\frac{1}{2} \leq \rho(1) \leq \frac{1}{2}$.

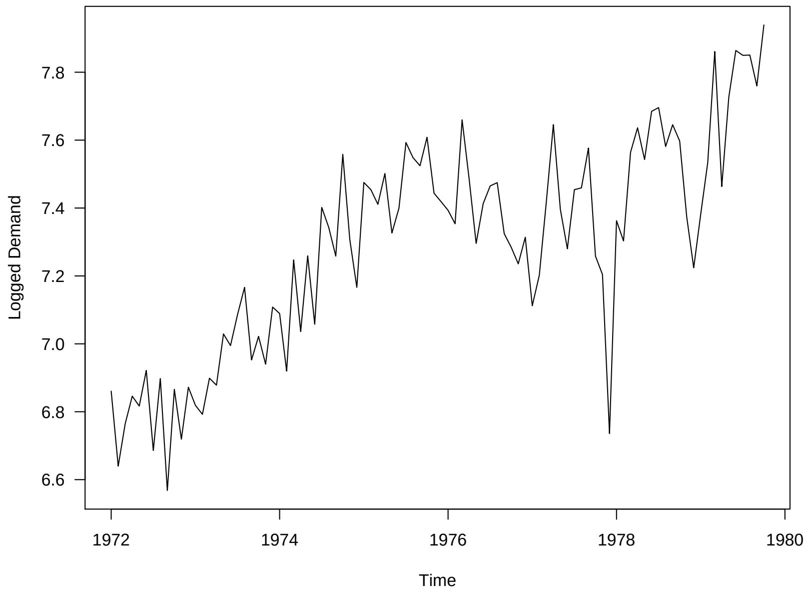
2. Your answers to this question should include a discussion of how general time series and statistical theory relates to this particular case. Short answers are not appropriate.
The question concerns a time series data set giving the monthly demand for repair parts large/heavy equipment in Iowa during the period 1972–1979.

(a) A graph of the series is shown below.



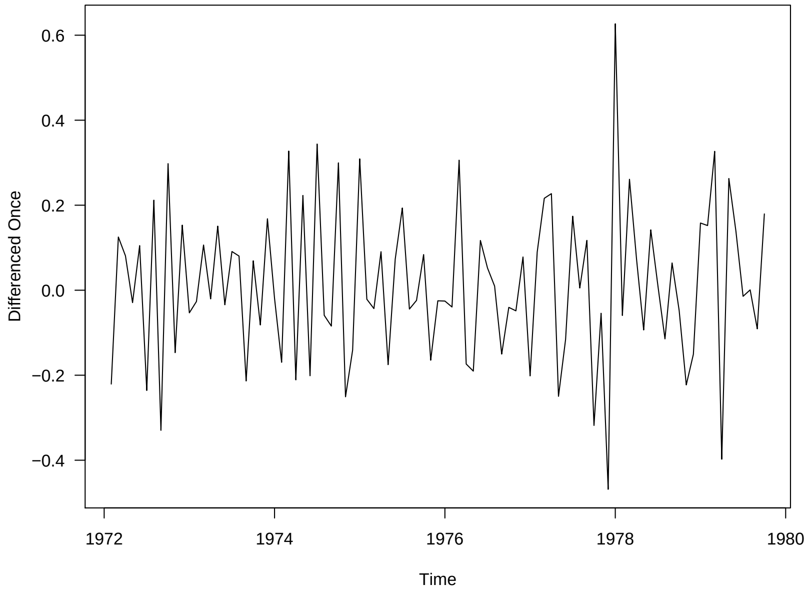
This graph indicates that it is appropriate to work with the logarithms of the original values. Explain why this is and indicate which competing transformations (if any) could have been tried.

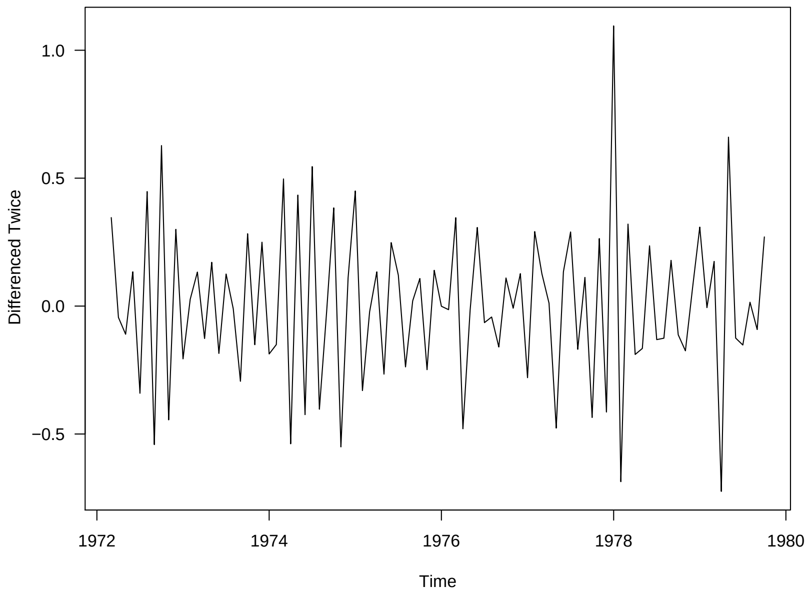
(b) A plot of logged values is shown below.



On the basis of this plot it was decided to work with a differenced variant of the series. Explain why this was and indicate why differencing is a sensible way to treat this particular problem (there should be some equations here).

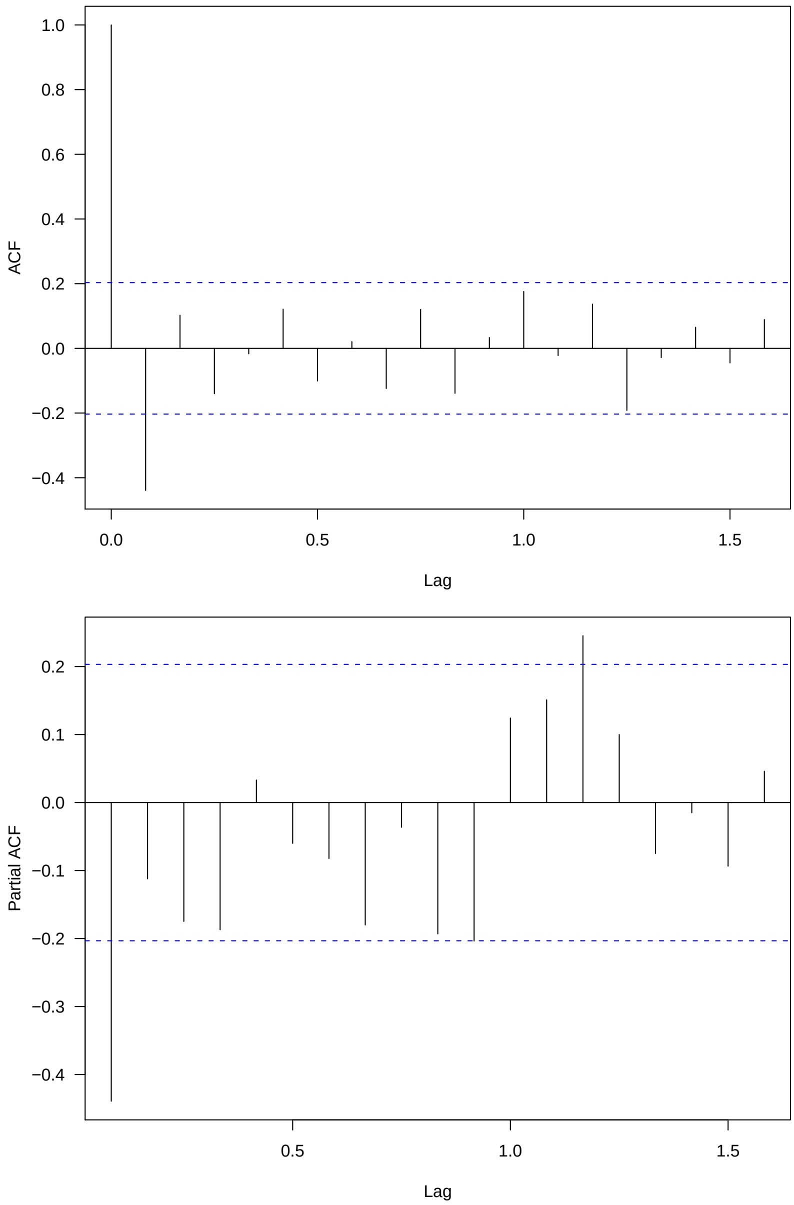
(c) The plots below show the first and second differences for the logged series.





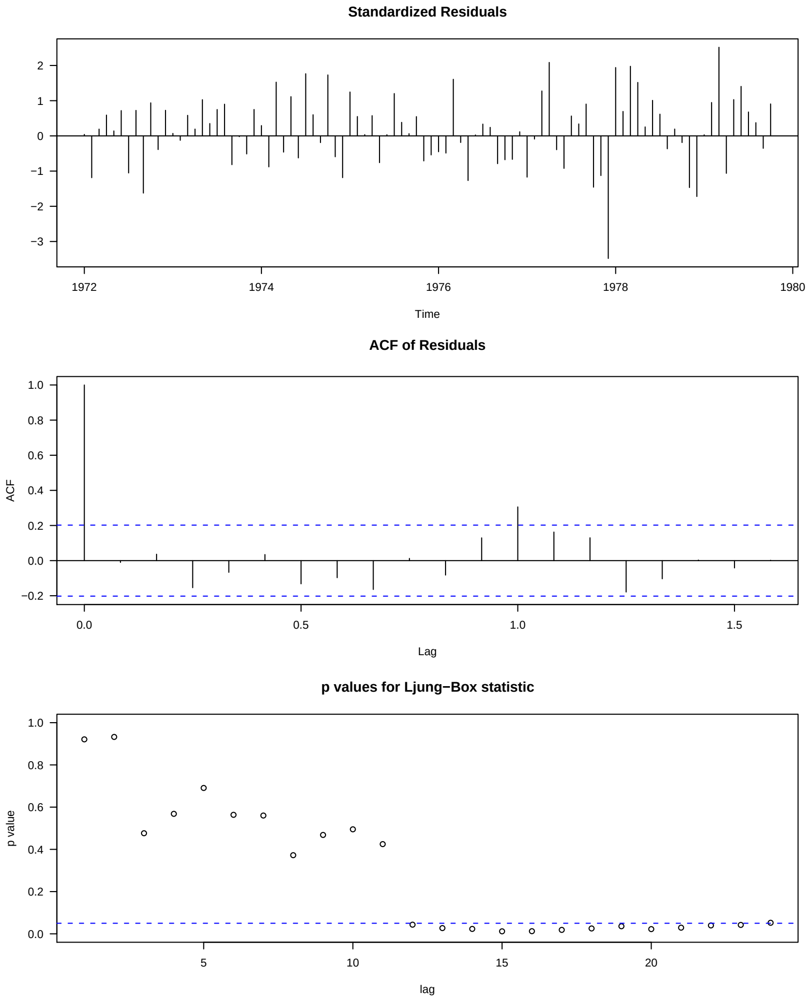
On the basis of these and the previous plots, explain what you think the right amount of differencing to apply to the series is.

(d) The following plots show the acf and pacf for a differenced version of the logged series.



On the basis of these plots, choose a model to be fitted to the logged data values (you will need to include differencing). Explain fully why you think the model you have chosen is appropriate.

(e) After fitting a model to the series, the following regression diagnostic plots were obtained.



Explain what each of these plots say about the quality of the model's fit.

(f) Taking into account the plots above, indicate how you would go about trying to improve the fit of the model.

3. (a) In this question suppose that $\mathcal{A}$ is a linear time invariant filter defined by coefficients $\{a(u)\}$.

(i) What does it mean for a filter to be *linear* and *time-invariant.*

(ii) Suppose that $Y(t)$ is obtained from $X(t)$ by filtering with $\mathcal{A}$. Write down the explicit formula relating $X(t)$ and $Y(t)$ in terms of the filter coefficients.

(iii) Write down the formula for the *transfer function* $A(\lambda)$ of $\mathcal{A}$.

(iv) Write down the definition of the *power spectrum* of a stationary, mixing time series.

(v) If $X(t)$ and $Y(t)$ are stationary time series with power spectra $f_{XX}(\lambda)$ and $f_{YY}(\lambda)$ write down the formula which relates $f_{XX}(\lambda)$ and $f_{YY}(\lambda)$.

(b) Suppose that $\{X(t)\}$ is a stationary time series with autocovariance function $c_{XX}(u)$ which satisfies the mixing condition

$$\sum_{u=-\infty}^{\infty} |c_{XX}(u)| < \infty.$$

(i) Define the *discrete Fourier transform* $\mathrm{d}_X^T(\lambda)$ of the T values $X(0), \ldots, X(T-1)$.

(ii) Show that $\mathrm{d}_X^T(-\lambda) = \overline{\mathrm{d}_X^T(\lambda)}$, and $\mathrm{d}_X^T(\lambda + 2\pi) = \mathrm{d}_X^T(\lambda)$.

(iii) Describe the asymptotic distribution of the discrete Fourier transform.

(iv) Define the *periodogram* $I_{XX}^T(\lambda)$ of the T values $X(0), \ldots, X(T-1)$.

(v) The periodogram is not a good estimator of the power spectrum of a stationary time series. Explain why not, and indicate how an improved estimator can be obtained from it.

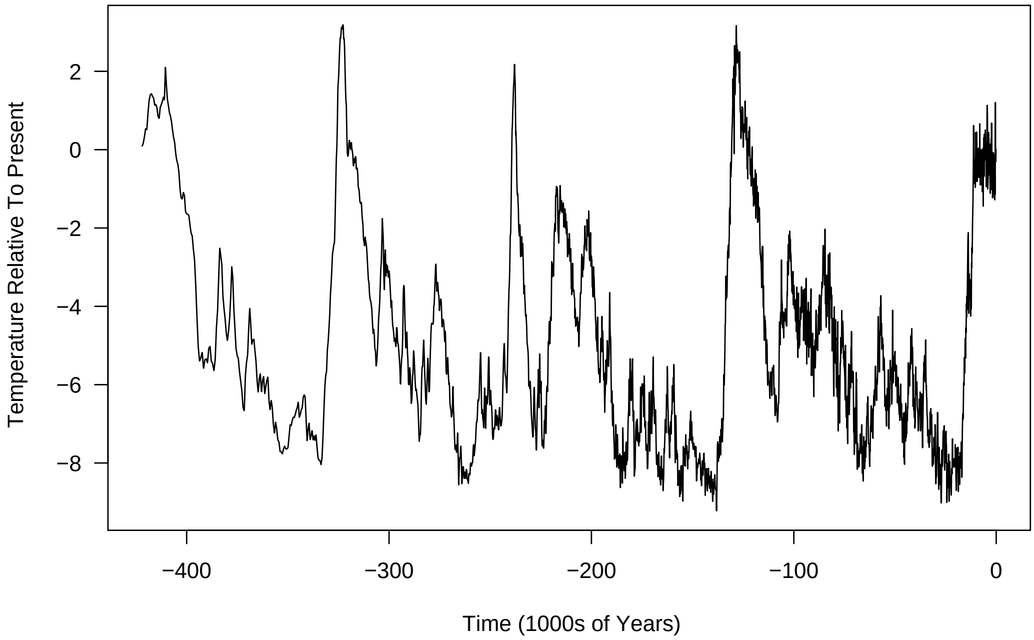
(c) (i) Derive the transfer function for the differencing filter defined by $Y(t) = X(t+1) - X(t-1)$, and compute its gain and phase function.

(ii) Compute the transfer function of the *Hann* filter with coefficents

$$a(u) = \begin{cases} 1/4 & u = -1 \\ 1/2 & u = 0 \\ 1/4 & u = +1 \end{cases}$$

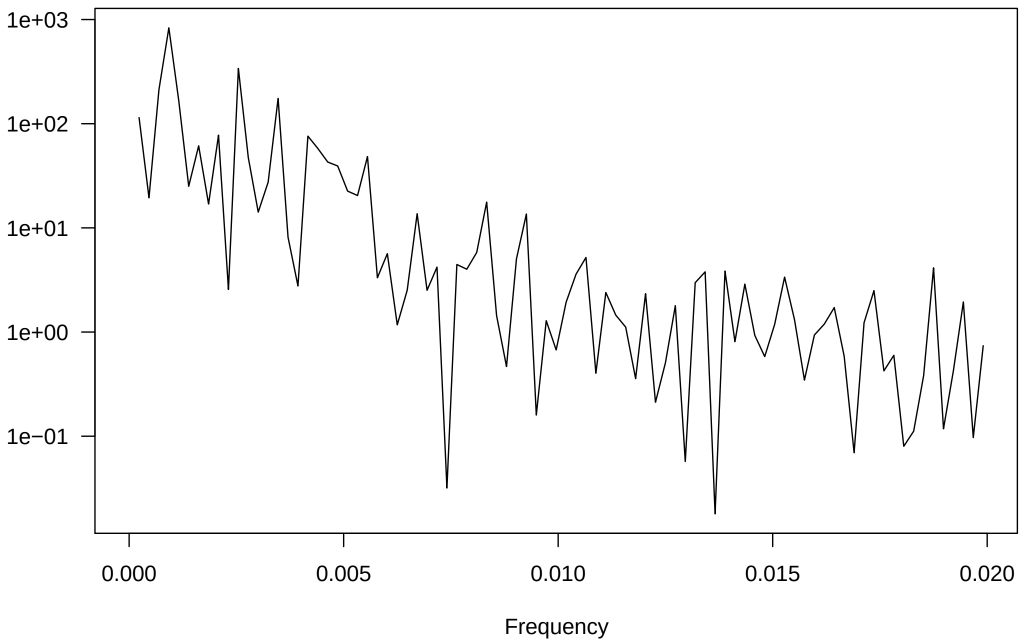
and describe the effect of filtering with these coefficients.

4. (a) The following figure shows a graph of temperature (relative to the present) as reconstructed from ice cores taken at Vostok research station in Antarctica. The temperatures are believed to reflect the temperature variation of the entire southern hemisphere and perhaps of the whole globe.

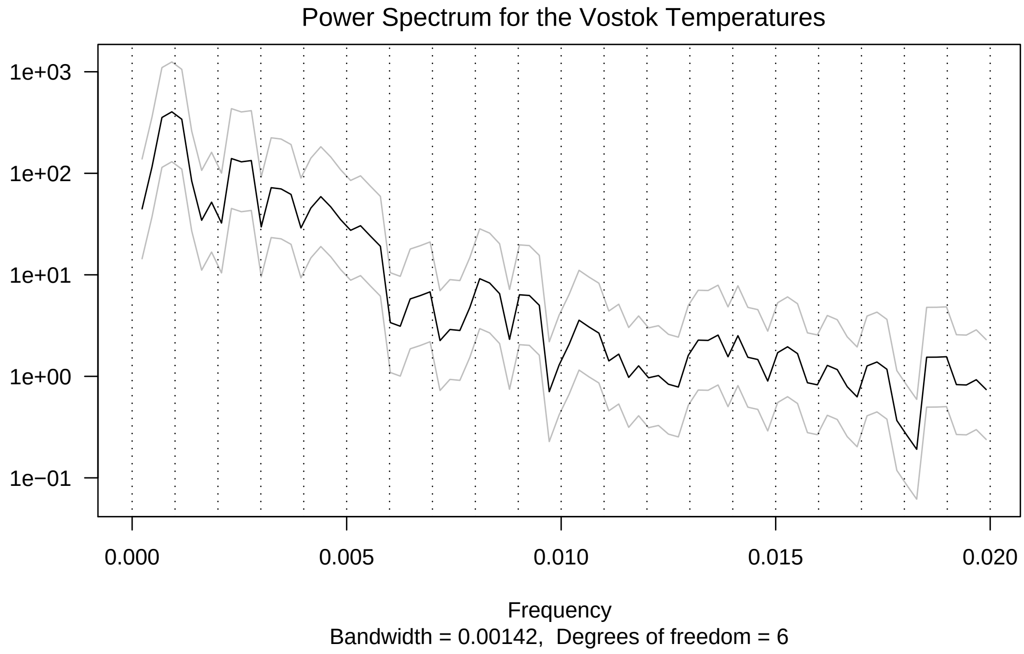


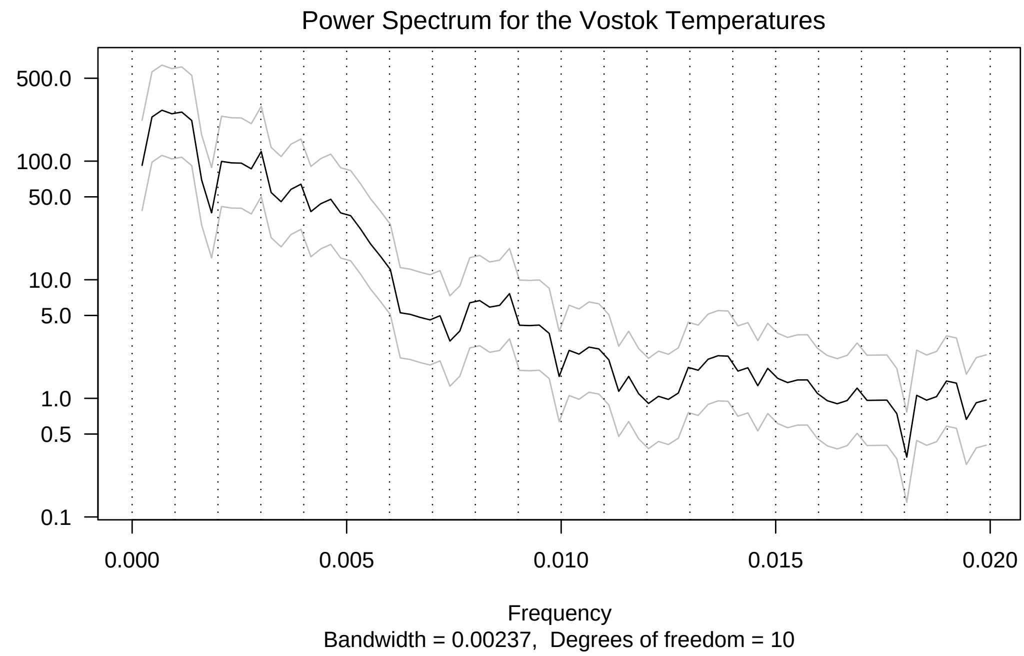
The graph shows a succession of "ice ages" separated by brief interglacial periods where temperatures equal or exceed those of the present time.

The record above has irregularly spaced time values. To carry out a time series analysis, the record was sampled at 100 year intervals to produce a regular time series. The periodogram of the resulting series is shown below.



The figures below show two power spectrum estimates for the Vostok temperature series. The estimates were obtained using a fixed-span smoother with different span values.





(i) The periodogram and spectra displayed earlier show frequency in cycles per unit time. How can this be converted into cycles per year?

(ii) Why would it **not** be appropriate to consider transformations such a logs and square roots for this series?

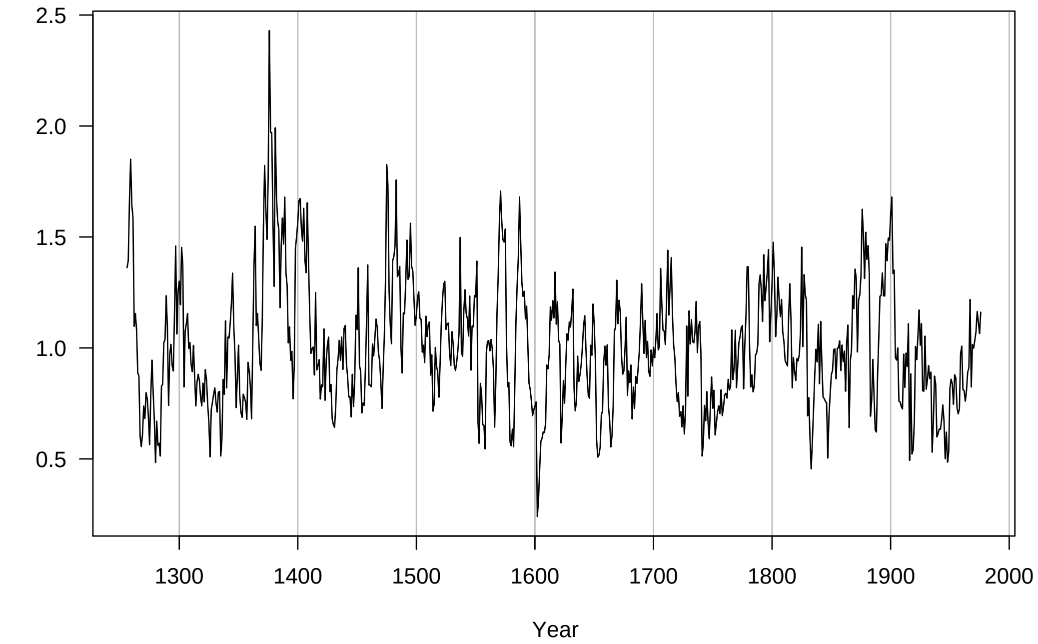
(iii) The power spectra above where obtained using fixed span smoothers. Identify the span used in each case.

(iv) The periodogram and spectra have been graphed on a log scale. Explain why this is appropriate.

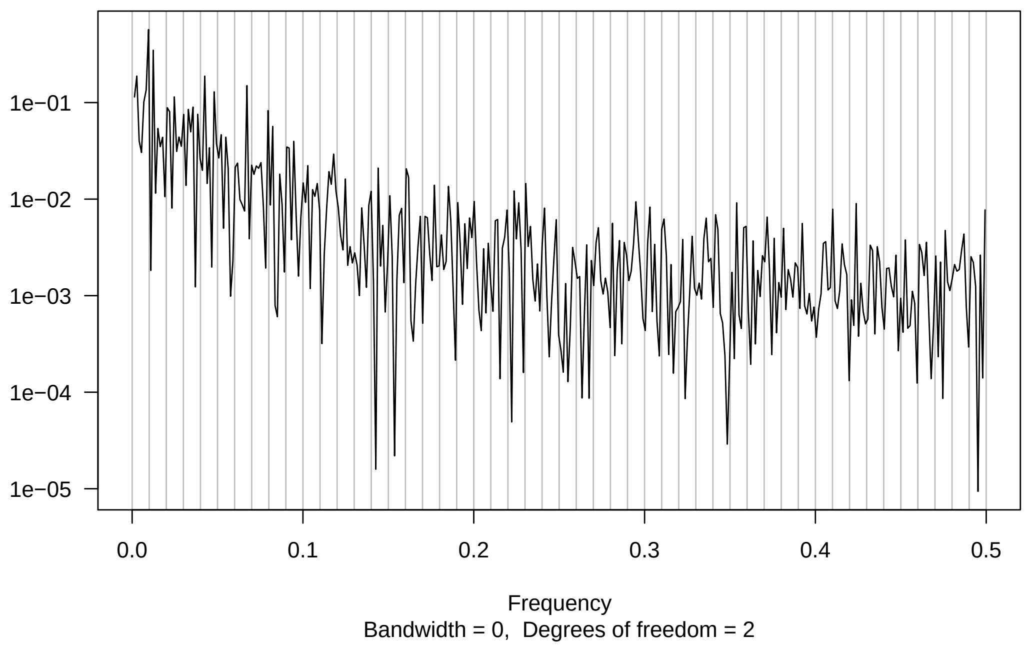
(v) Does the periodogram show evidence of periodicities in the original series? If there are periodicities, identify the corresponding frequencies.

(vi) Describe any features of this spectrum which might be considered unusual.

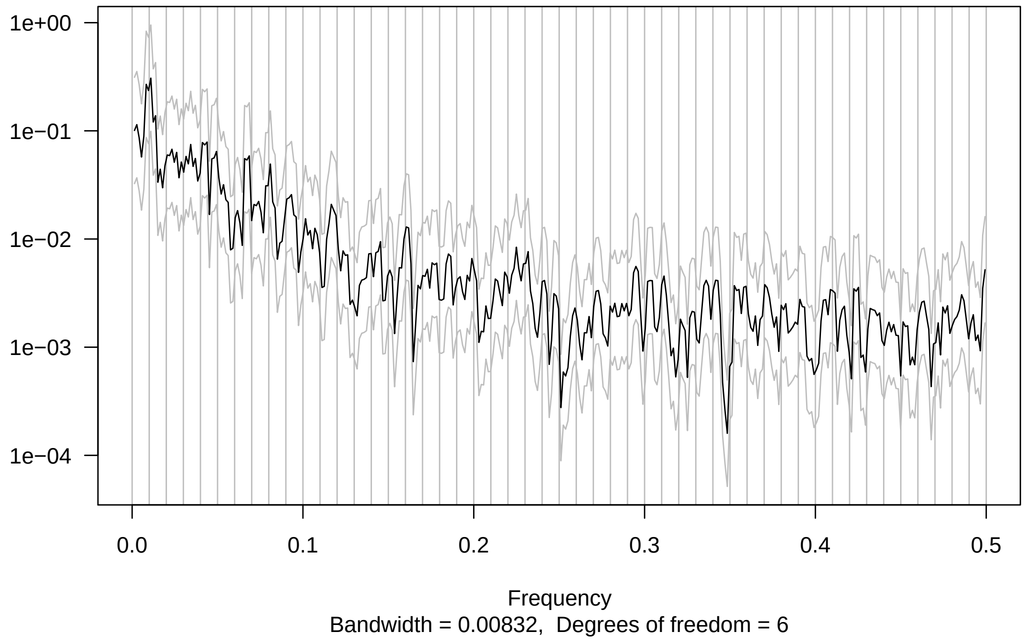
(b) Tree rings show how much a tree grows in each year of its life. They are of interest because they provide a historical record of the weather during the lifetime of the tree. The following plot shows the record of growth rings (in unspecified units) from a tree at Takapari, New Zealand, during the period 1256–1976.

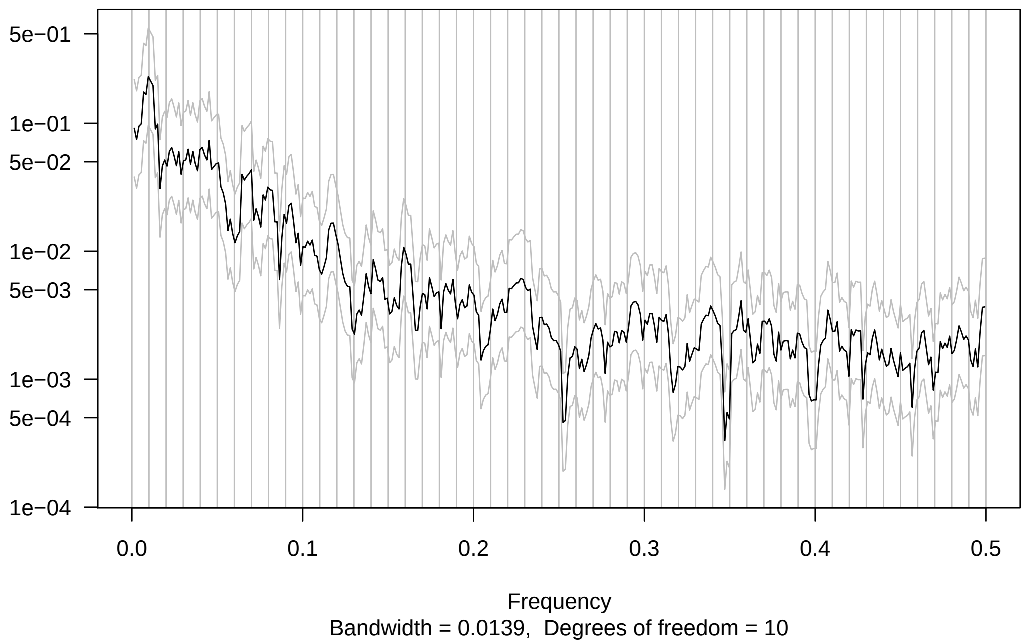


The following plot shows the periodogram of the tree ring series.



The following plots show power spectrum estimates for the tree ring series.





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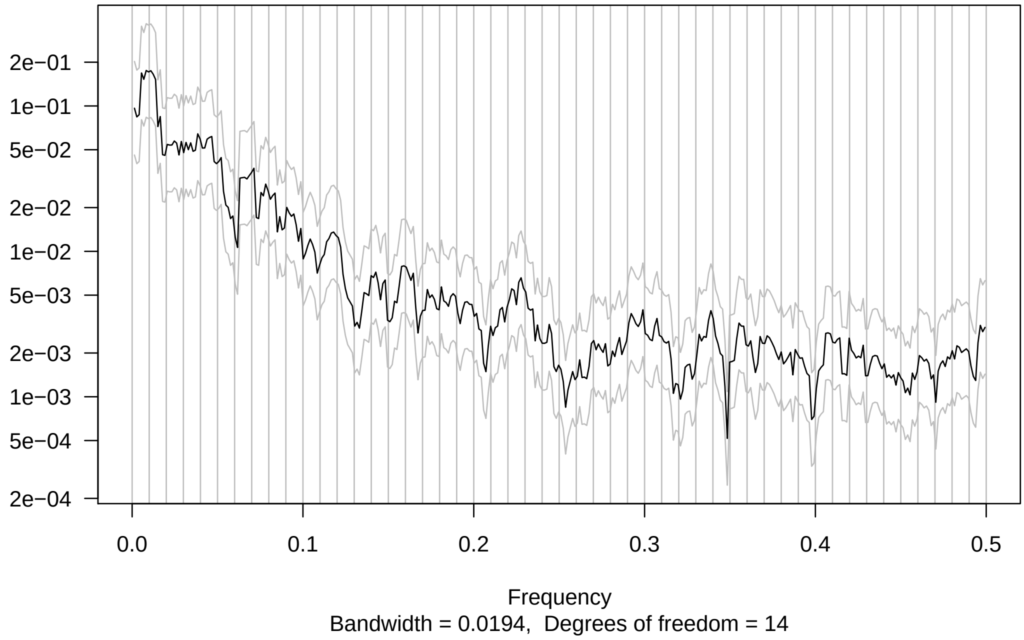

Explain and interpret this analysis.