THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2005
Campus: City

STATISTICS

Time Series Analysis

(Time allowed: TWO hours)

NOTE: Attempt all FOUR questions.
All questions are worth equal marks.
You should allot equal time for answering each question.

1. (a) Define what it means for a time series to be *stationary* [2 marks]

(b) What does it mean for a time series to be *causal*. [2 marks]

(c) Define, precisely, what it means for a time series Y_t to be an AR(1) series. [2 marks]

(d) What condition will ensure that an AR(1) series will be stationary and causal. [2 marks]

(e) Show that the autocovariance function of an AR(1) series is given by

$$\gamma(u) = \frac{\sigma_\varepsilon^2 \phi^{|u|}}{1-\phi^2}, \qquad \text{for } u = 0, \pm 1, \pm 2, \ldots$$

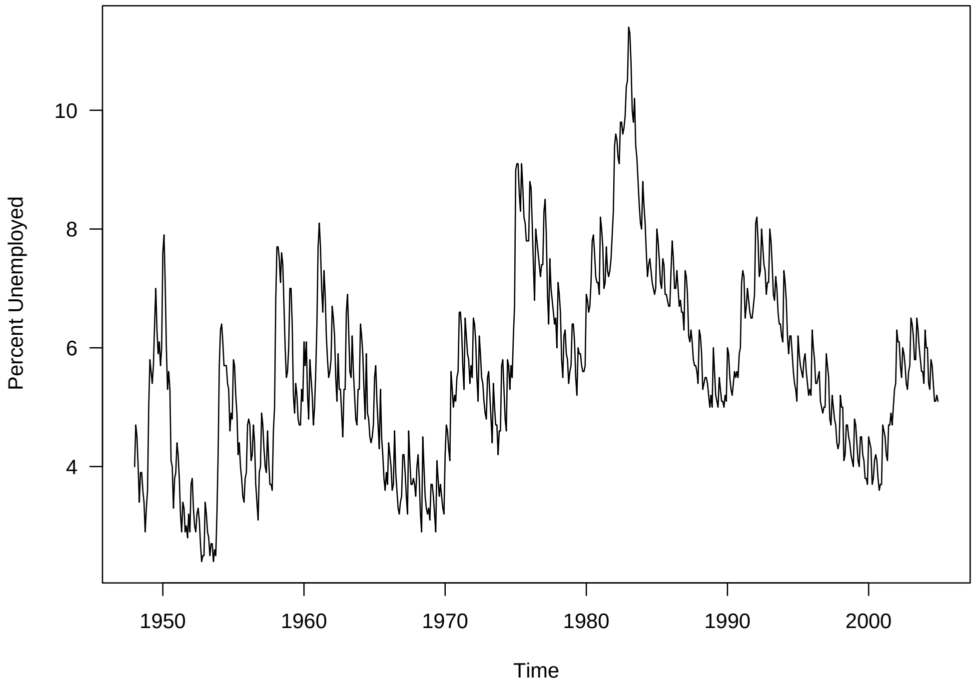
where ϕ is the parameter of the AR(1) series. [5 marks]

(f) Suppose that Y_t is a stationary, causal AR(1) series. Describe, as exactly as you can, the type of ARMA series which is produced by taking simple differences of Y_t. [5 marks]

(g) Compute the variance of the differenced series of part 1f above. [5 marks]

(h) Write down the *operator* form of the general ARIMA(p,d,q) series. [2 marks]

2. The figure below shows the United States monthly unemployment rate for the years 1948 – 2004.



(a) In order to forecast the series, it was decided to difference the series twice; once at lag 1 and once at lag 12. Explain why such differencing is required and explain how the decision to difference with these particular lags may have been arrived at. [3 marks]

(b) The two plots on page 3 show the estimated acf and pacf for a differenced version of the unemployment series. Give a detailed explanation of how these plots can be used to suggest a model for the differenced series and write down the model you feel is appropriate here, explaining why you think it is appropriate. [5 marks]

(c) An ARMA$(2, 1, 2) \times (2, 1, 2)_{12}$ model was fitted the series. Write down in mathematical terms what this means (i.e. give an equation for the model). [3 marks]

(d) The coefficients obtained by fitting an ARMA$(2, 1, 2) \times (2, 1, 2)_{12}$ are shown below. Based on these results, describe how you would go about searching for better fitting models. [4 marks]

```
Coefficients:
           ar1      ar2      ma1      ma2
        0.8045  -0.0763  -0.6917   0.1693
s.e.    0.0129   0.0181   0.0193   0.0293
          sar1     sar2     sma1     sma2
       -0.2639  -0.0142  -0.4980  -0.1790
s.e.    0.0183   0.0149   0.1014   0.0472

sigma^2 estimated as 0.0596
log likelihood = -11.04
aic = 40.08
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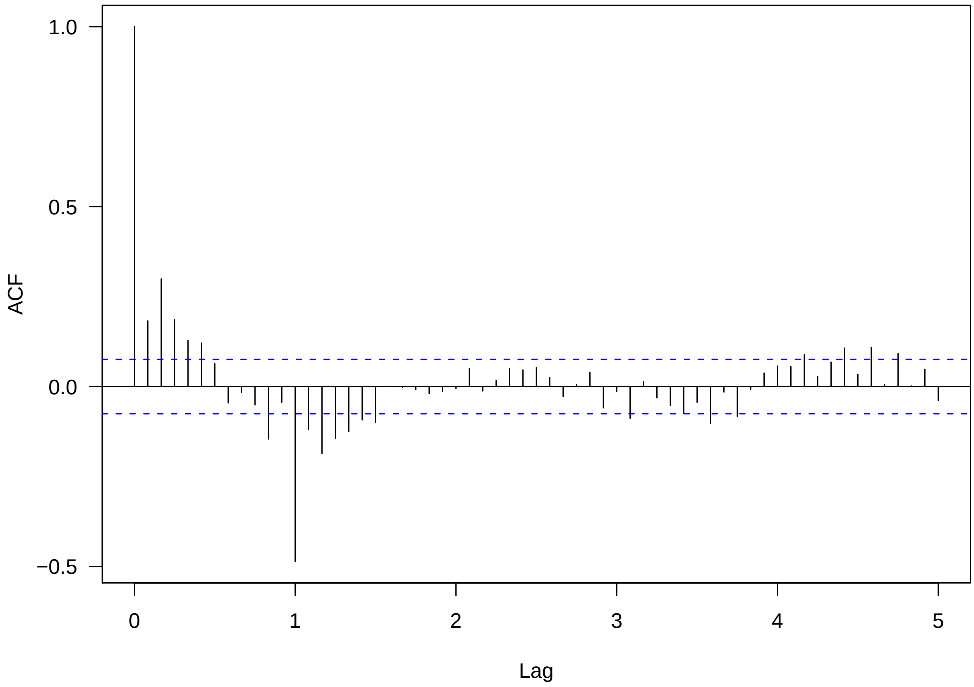
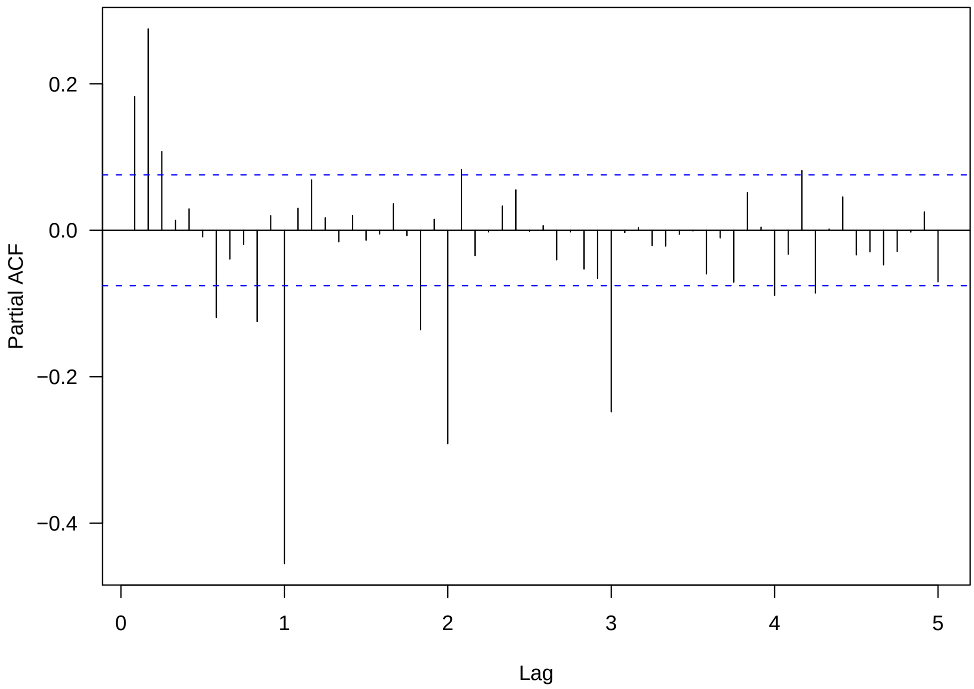
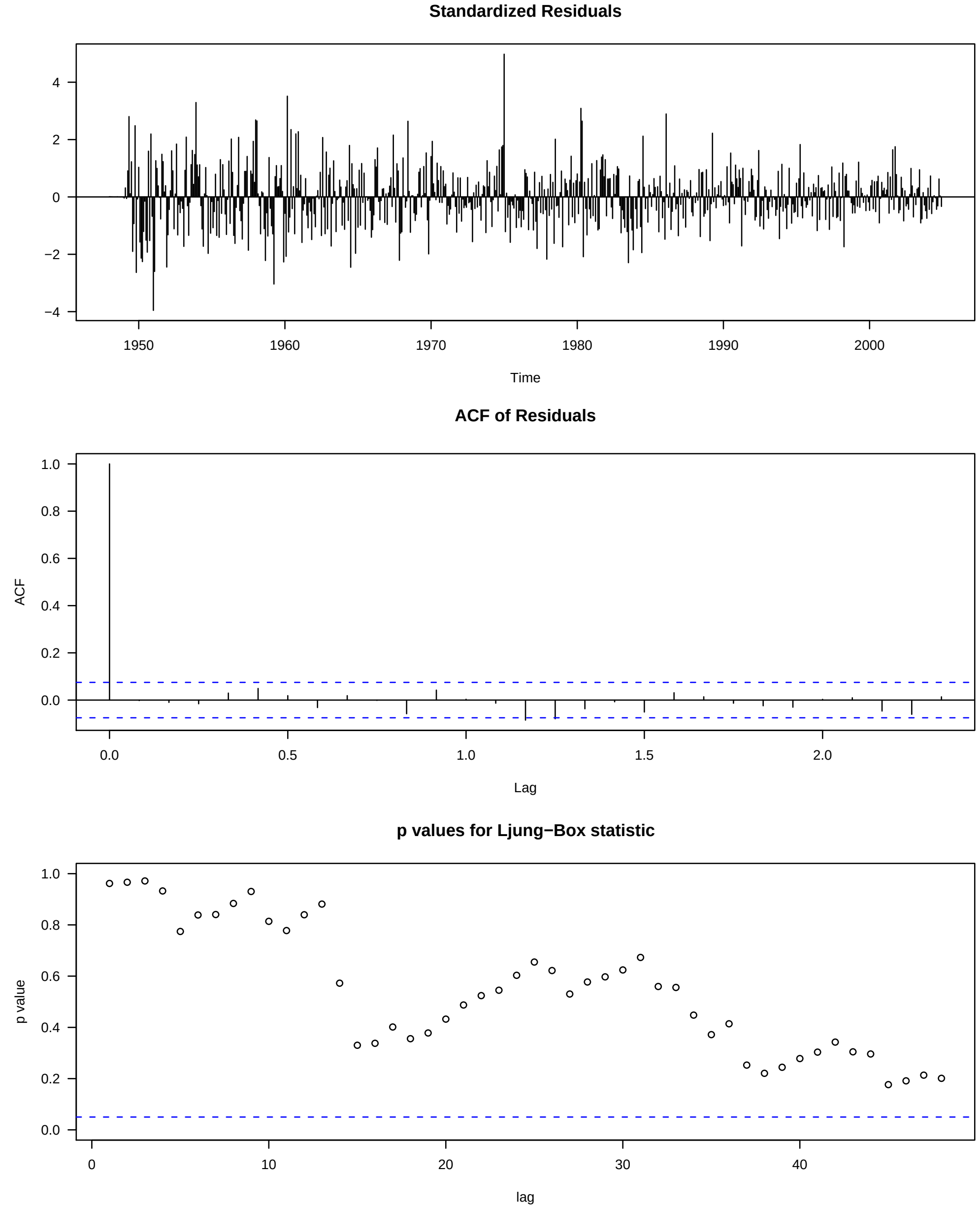


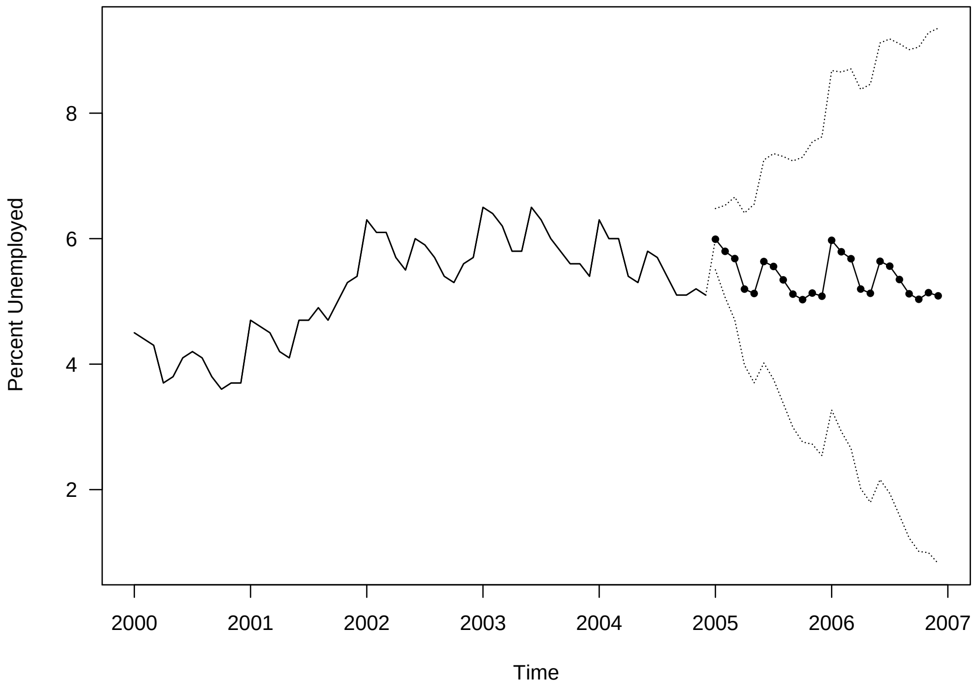


The acf and pacf functions for the twice-differenced U.S. unemployment series.



(e) The plots above show diagnostics for the residuals obtained after fitting a model to the unemployment series. Describe the kinds of problems that each of the plots can display. [3 marks]

(f) Are there any features present in the plots which indicate that there might be problems with the fit? [4 marks]



(g) The figure above shows forecasts (with two standard error limits) which were generated for the unemployment series. How would you interpret this plot? [3 marks]

3. (a) Consider the symmetric differencing filter described by

$$Y(t) = \frac{X(t+1) - X(t-1)}{2}.$$

(i) Write down the *impulse response* (i.e. the $a(u)$ function) for this filter. [2 marks]

(ii) Show that the *transfer function* for the filter is [5 marks]

$$A(\lambda) = i\sin(\lambda), \qquad \text{for } \lambda \in [0, \pi].$$

(iii) Describe, in general terms, the effect that this filter will have on time series it is applied to. [2 marks]

(b) Suppose that $X(t)$ is a time series with autocovariance function $\gamma(u)$ and suppose that $\gamma(u) \to 0$ very quickly as $u \to \infty$.

(i) Write down the definition of the power spectrum, $f_{XX}(\lambda)$, of $X(t)$. [2 marks]

(ii) Suppose that $Y(t)$ is obtained by applying a linear time invariant filter to $X(t)$. Write down the relationship between the power spectrum of $Y(t)$ and the power spectrum of $X(t)$. [2 marks]

(iii) What is the (approximate) distribution of the periodogram $I^T_{XX}(\lambda)$. [3 marks]

(iv) Indicate what the problems are with the periodogram as an estimate of the power spectrum, and indicate how these problems can be addressed. [4 marks]

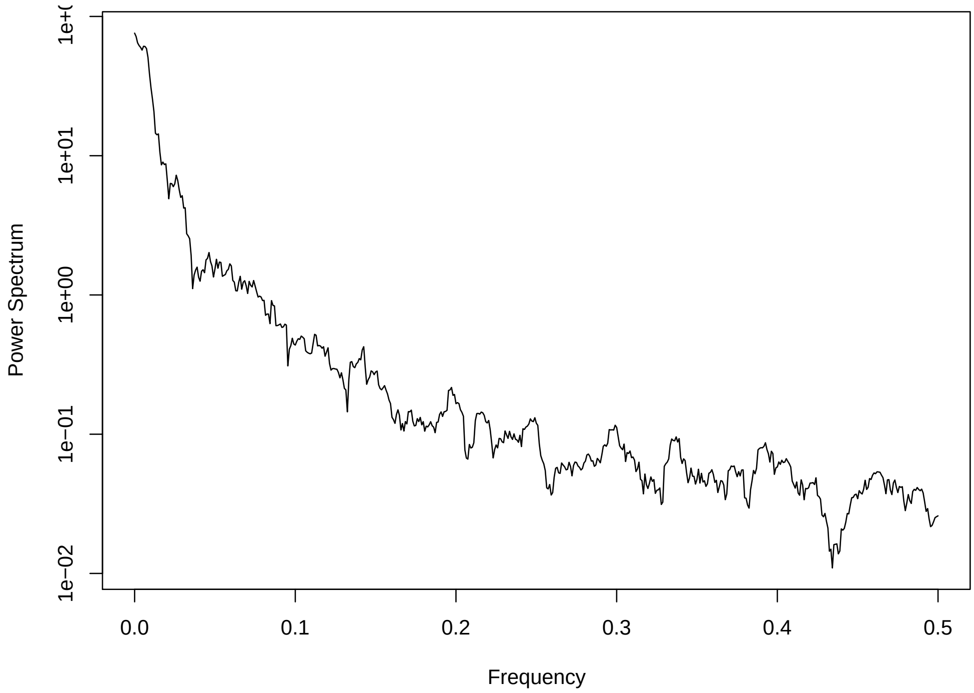
(c) Suppose that it is suspected that there is an imperfect linear time-invariant relationship between a series $Y(t)$ and a series $X(t)$.

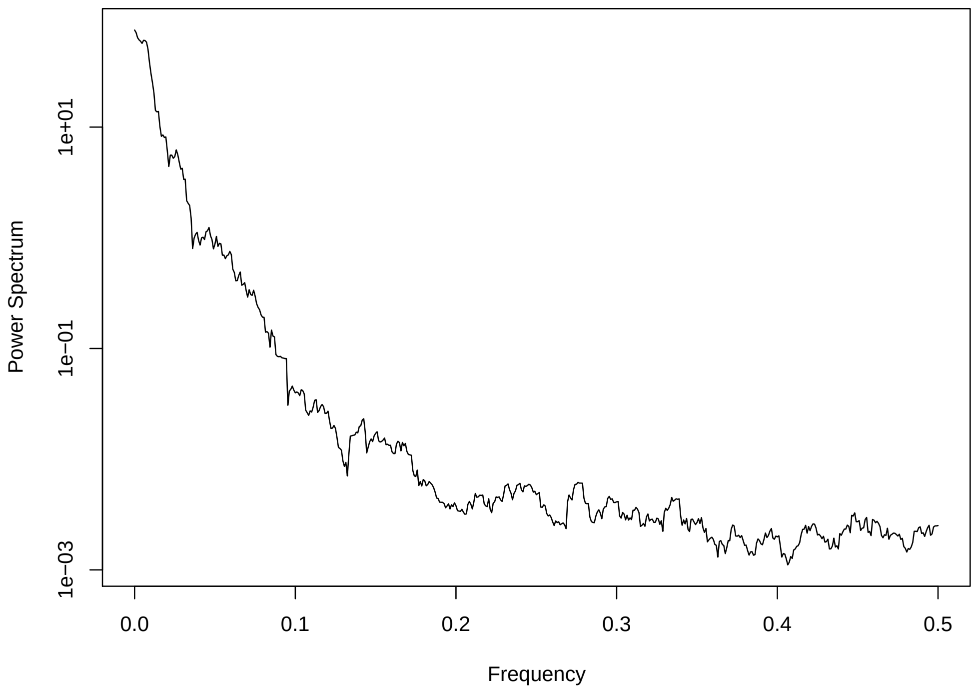
(i) Write down an appropriate model which can be fitted in this case, describing carefully what the parts of the model mean. [2 marks]

(ii) Describe what is meant by the coherence between two time series and how it is interpreted. [3 marks]

4. (a) The following seven plots show a cross-spectral analysis carrried out for two (simulated) time series. Explain, in general terms,what kind of information each type of plot presents, and interpret the particular pattern present in these particular plots.

What is the nature of the relationship between the two series? [13 marks]





Power spectra for the X series (top) and Y series (bottom).

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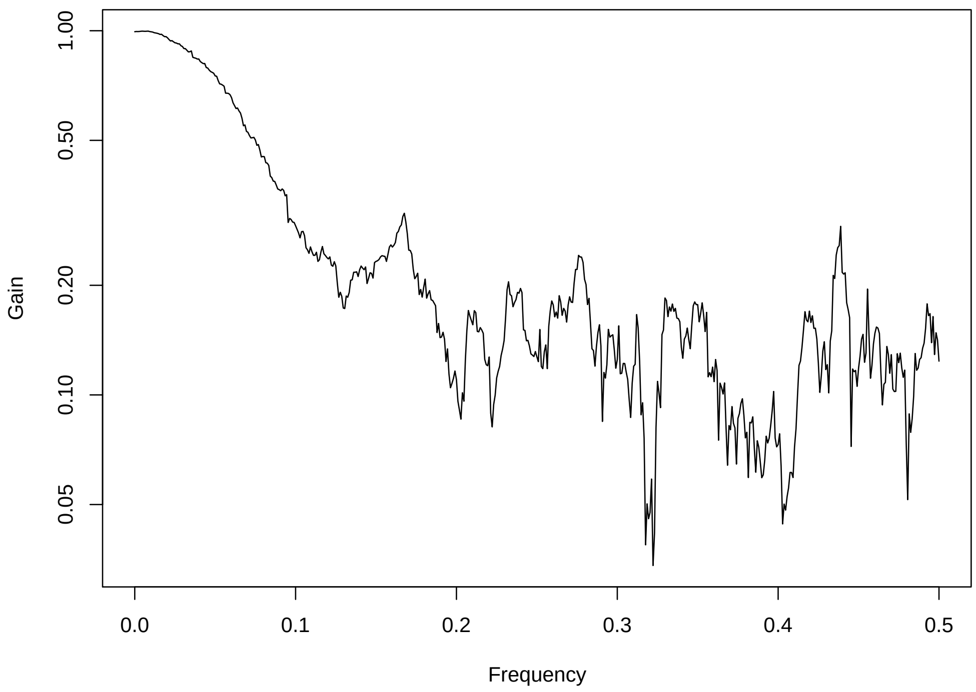

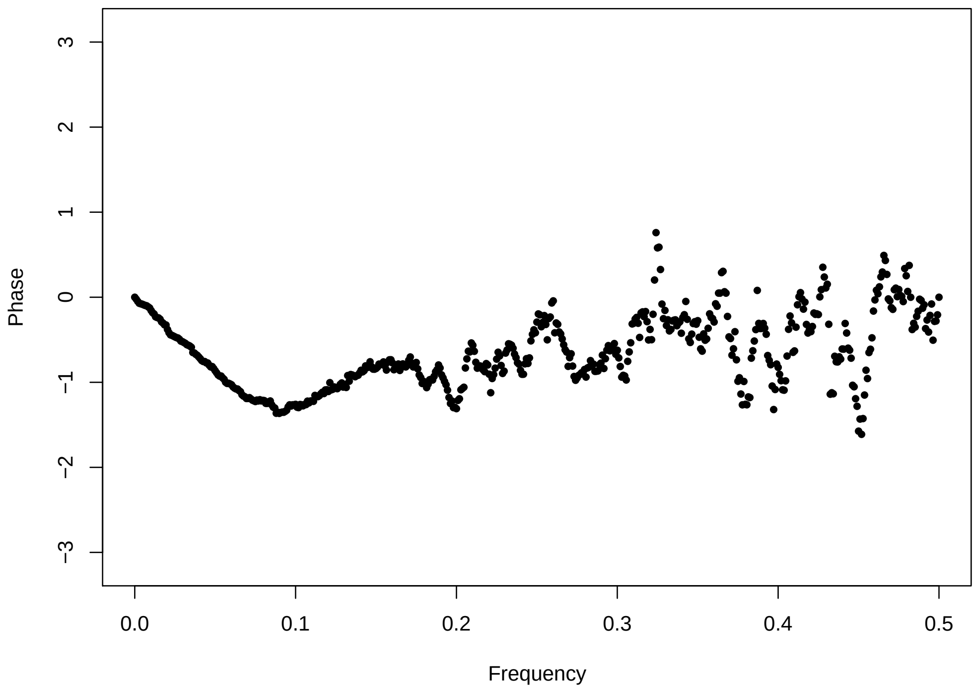

The gain (top) and phase (bottom) of the fitted relationship.

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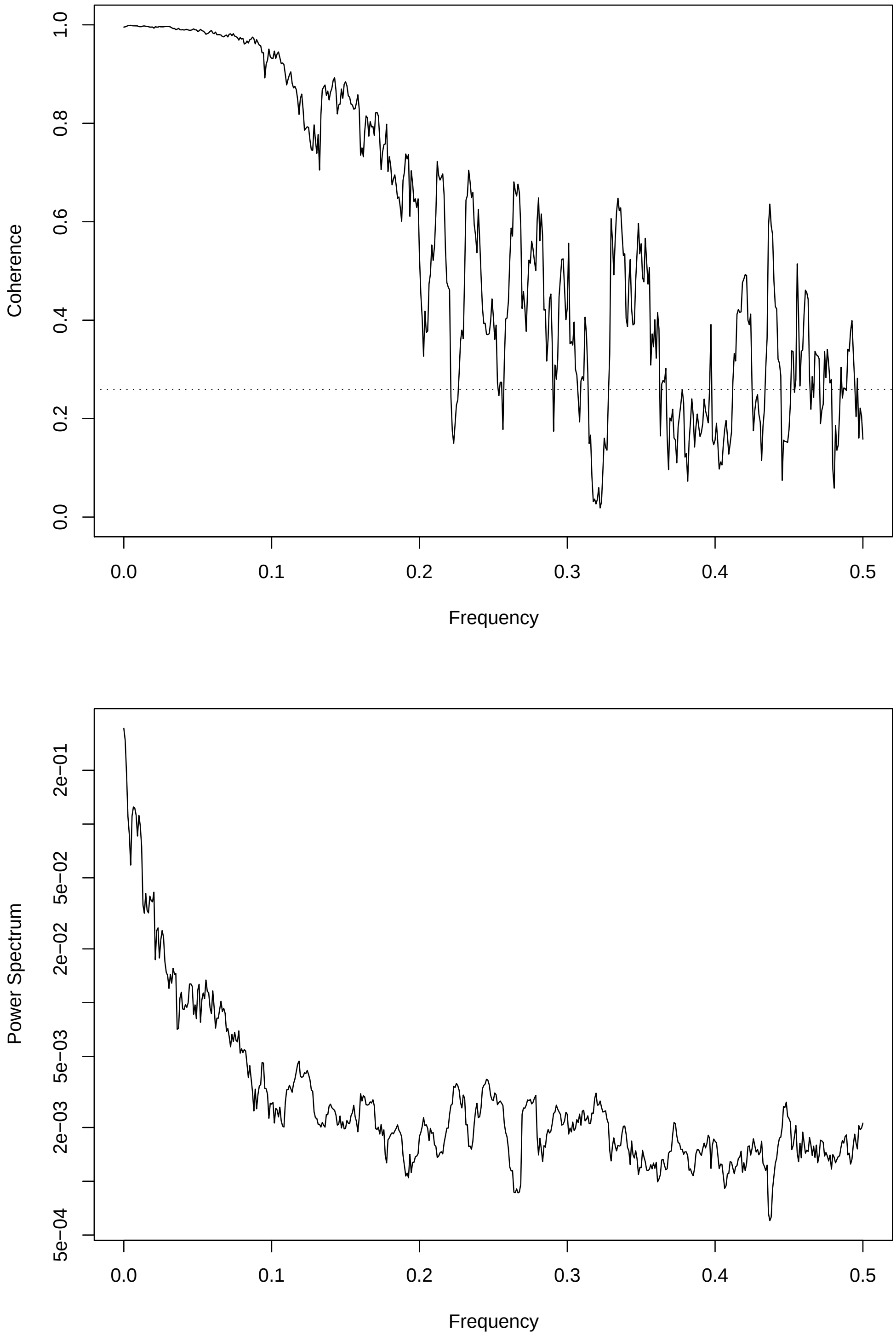
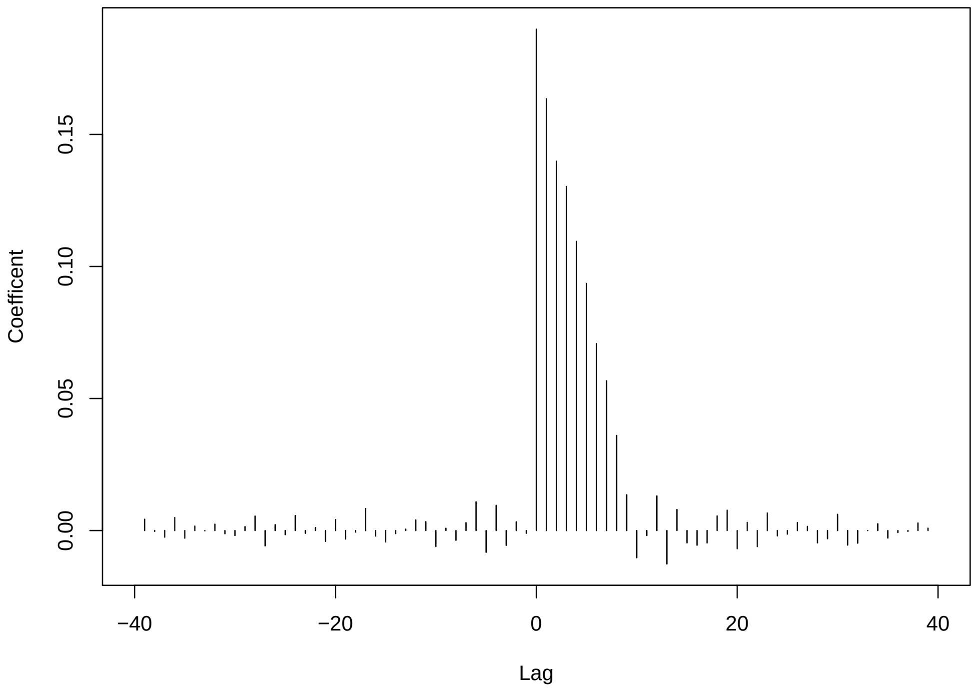


The coherence and residual spectrum for the fitted relationship.



The impulse response for the fitted relationship.

(b) The figures on the next page show a series of observations of the brightness of a star made on 600 successive nights by Isaac Newton, together with the periodogram for the same observations. Set down a mathematical model which explains the two plots. [12 marks]

Hint: it looks like each of the plots suggests a different model, but simple trigonometric identities show that the models are the same. You may or may not find the following identities useful.

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

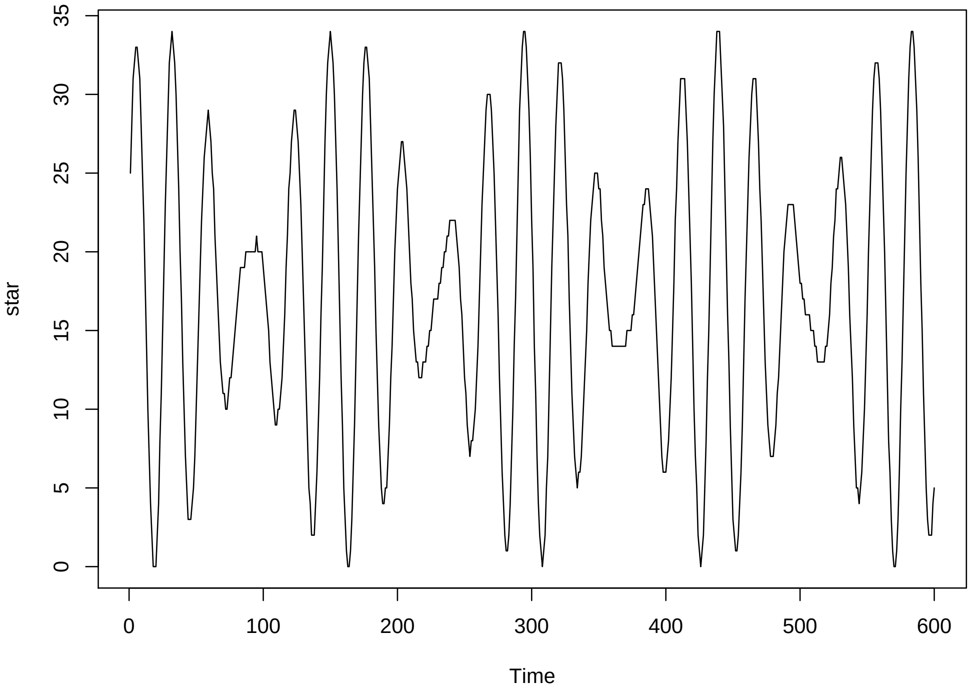
$$\sin(\theta + \phi) = \cos\theta\sin\phi + \sin\theta\cos\phi$$

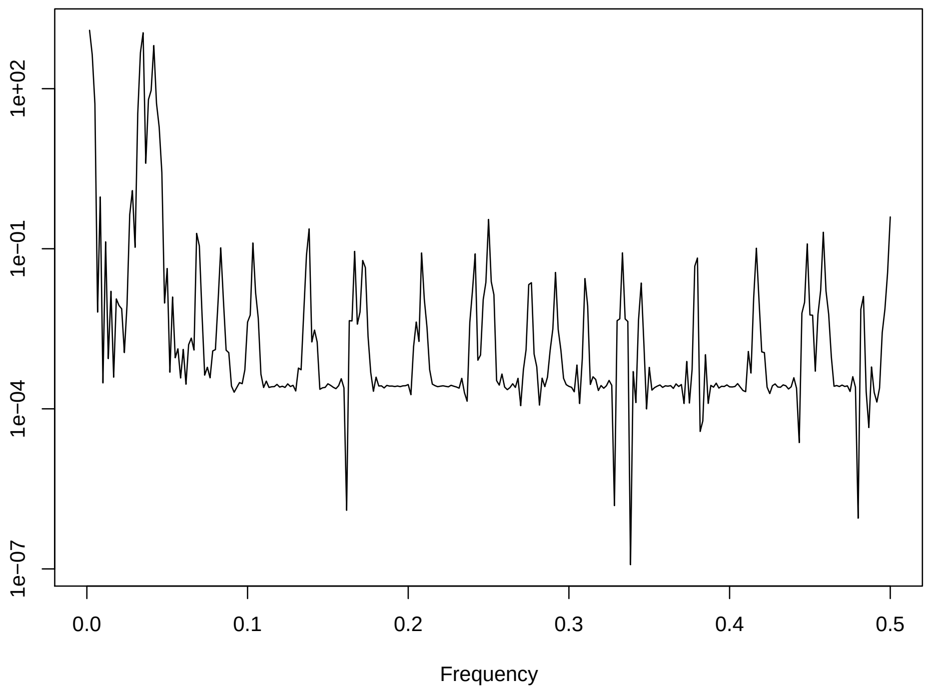
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$2\cos\theta\cos\phi = \cos(\theta - \phi) + \cos(\theta + \phi)$$

$$2\sin\theta\sin\phi = \sin(\theta - \phi) - \sin(\theta + \phi)$$





Star brightness observations (top) made by Isacc Newton and their periodogram (bottom).